De-complexifying radiation belt electron observations with adiabatic coordinates

Scot R. Elkington
LASP, University of Colorado

RBSP Science Working
Group Meeting
Boulder, CO
September 1, 2010

Describing the radiation belts

The radiation belts may be completely characterized at a point in time by its *distribution function:*

$$f = f(x, y, z, p_x, p_y, p_z)$$

Also referred to as the *phase space density*, f gives the number of particles in a volume $(x+dx, y+dy, z+dz)$, with momenta between $(p_x+dp_x, p_y+dp_y, p_z+dp_z)$.

The flux in a region of space may be related to the distribution function through

$$f = \frac{j}{p^2}$$

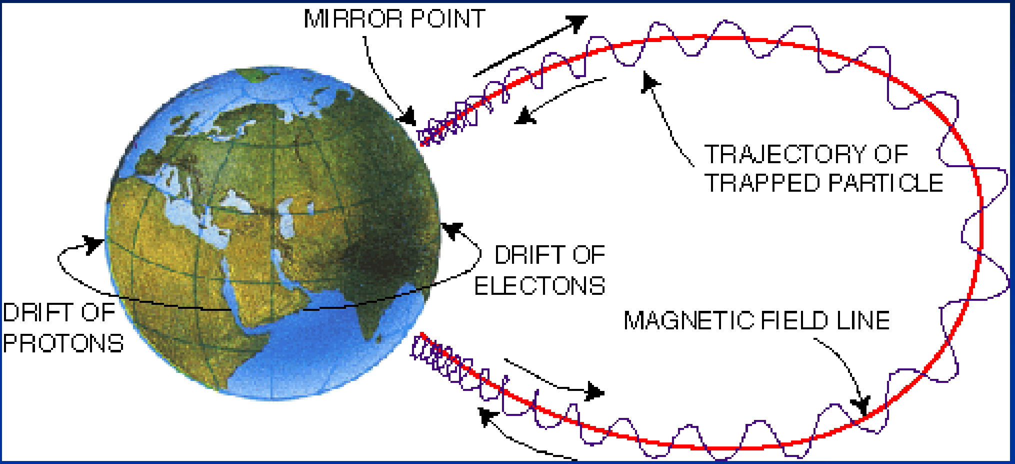
Adiabatic invariants

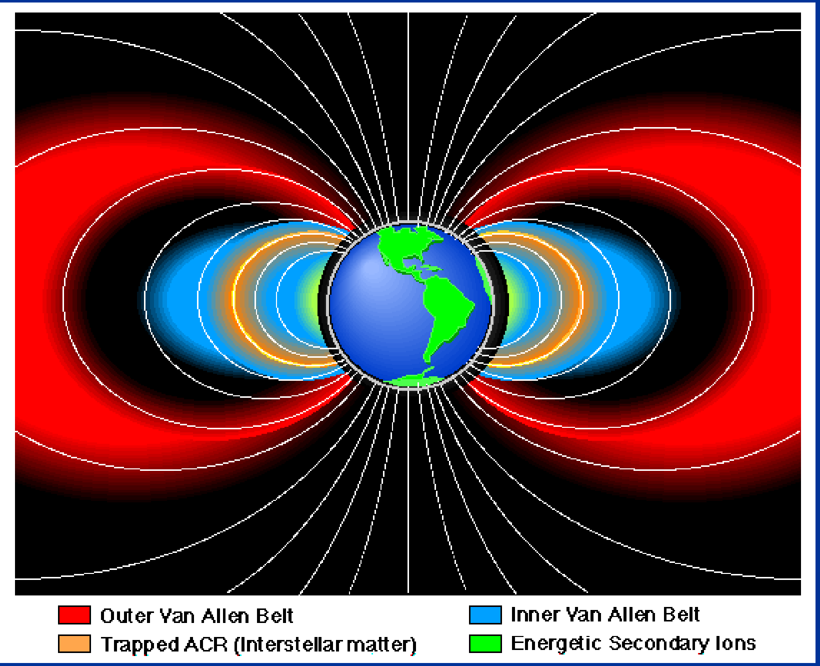
via the Hamilton-Jacobi action integrals, there is associated with each motion is a corresponding *adiabatic invariant:*

•Gyro: $M=p^2/2m_0B$

•Bounce: $J=pI(B)$

•Drift: Φ





- M: perpendicular motion
- K: parallel motion ($\sim J/M^{1/2}$)
- L: radial distance of eq-crossing in a dipole field ($=1/\Phi$).

If the fields guiding the particle change slowly compared to the characteristic motion, the corresponding invariant is conserved.

S. Elkington, GEM 2003

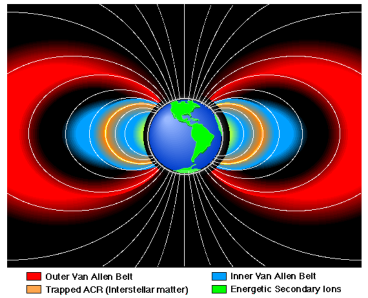
The distribution function and the adiabatic invariants

The distribution function may equivalently be written in terms of the invariants and corresponding phase:

$$\begin{aligned} f &= f(x, y, z, p_x, p_y, p_z) \\ &= f(M, K, L, \phi_1, \phi_2, \phi_3) \end{aligned}$$

If the distribution is uniform in phase (e.g. uniform ϕ_3 ⇨ no drift bunching in L), then the phase space density taken at a point may be considered the same at all points corresponding to the same M, K, and L.

$$\bar{f} = \bar{f}(M, K, L)$$



S. Elkington, GEM 2003

Transport and Fokker-Planck

The evolution of the phase space density is given by the Fokker-Planck equation:

$$\frac{d\bar{f}}{dt} + \frac{1}{\Im}\sum_i \frac{\partial}{\partial J_i}\left[\Im\left\langle\frac{dJ_i}{dt}\right\rangle\bar{f}\right] = \frac{1}{\Im}\sum_{i,j}\frac{\partial}{\partial J_i}\left(\Im D_{ij}\frac{\partial\bar{f}}{\partial J_j}\right) - \frac{\bar{f}}{\tau} + S$$

Sources

Coherent terms (e.g. friction)

Stochastic terms (e.g. diffusion)

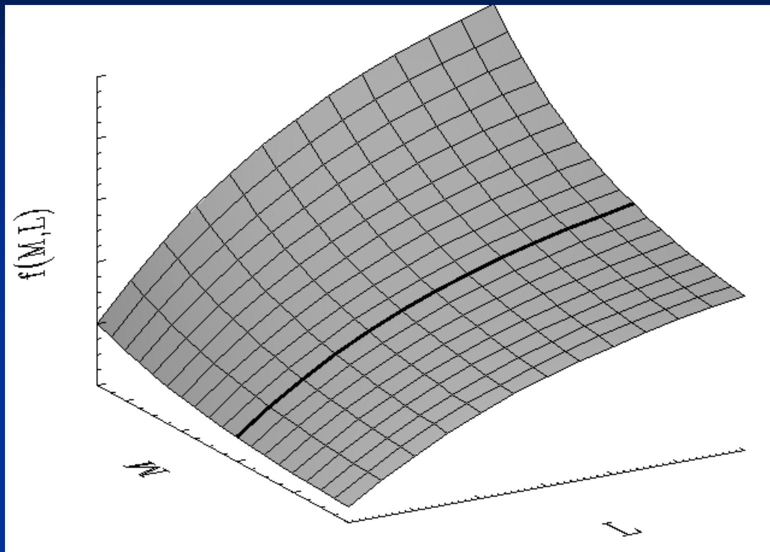
Losses

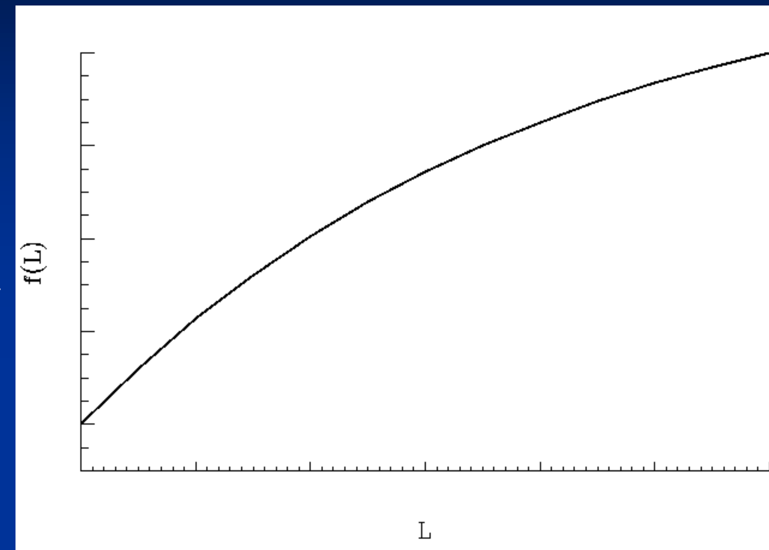
For example, to quantify a process that leads only to diffusion in L, we would write

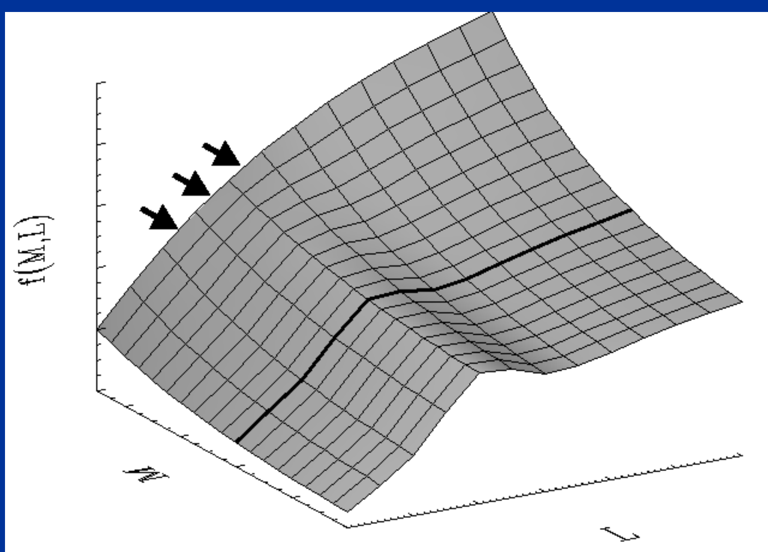
$$\frac{d\bar{f}}{dt} = L^2\frac{\partial}{\partial L}\left(\frac{1}{L^2}D_{LL}\frac{\partial\bar{f}}{\partial L}\right) - \frac{\bar{f}}{\tau}$$

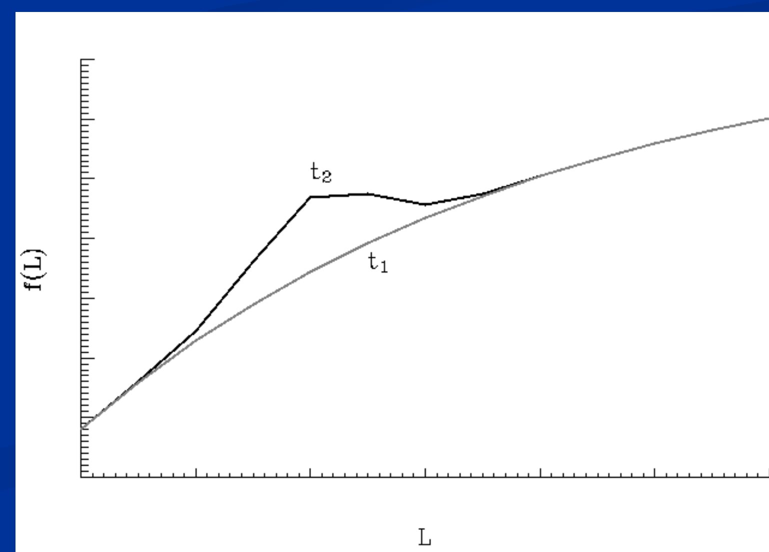
(M. Schulz, AGU Monograph 97, 1996)

Transport in M, K: local heating

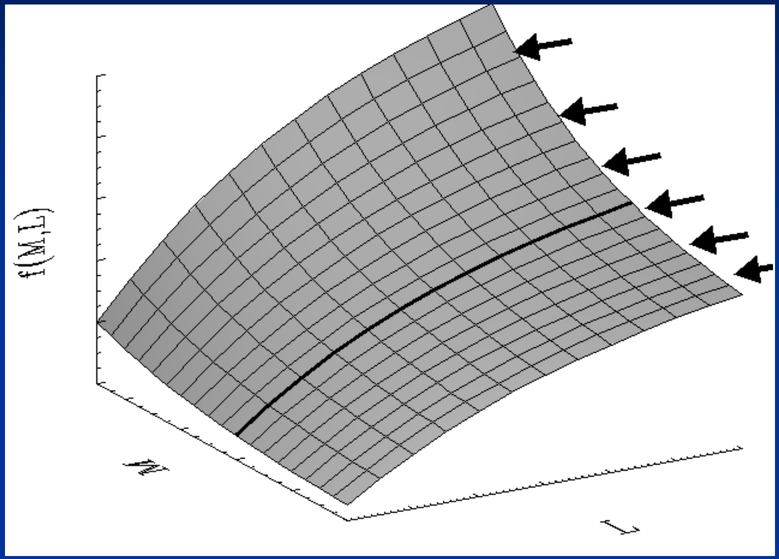


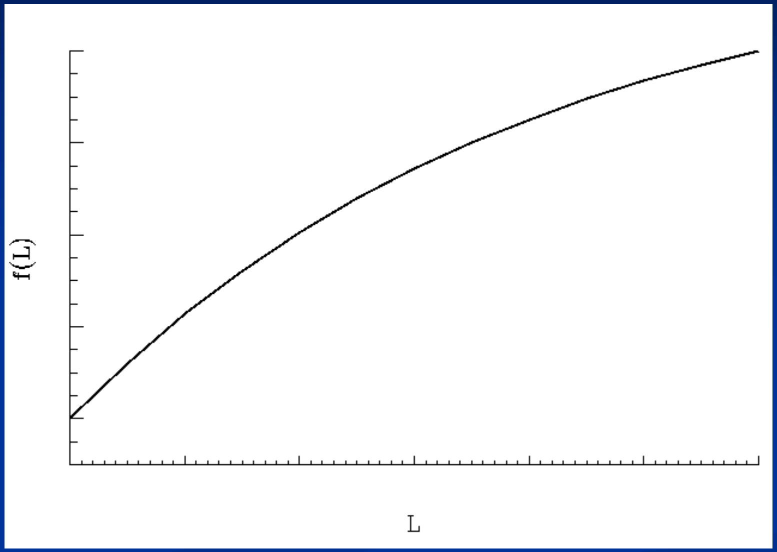


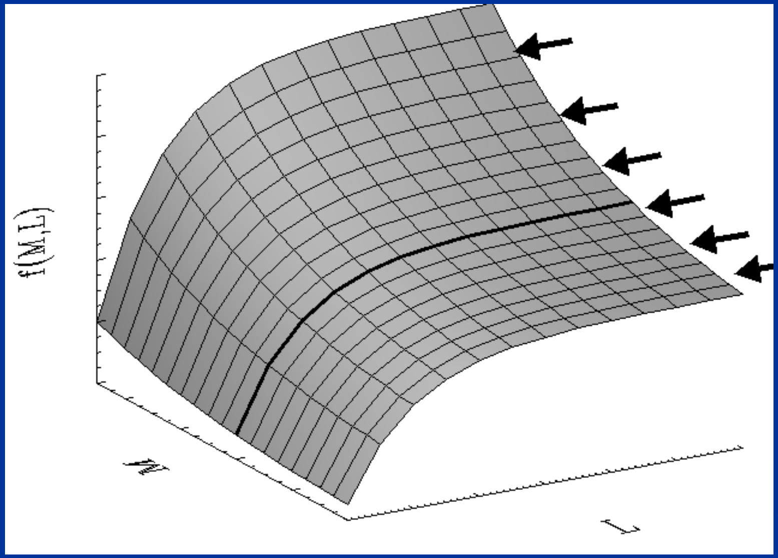


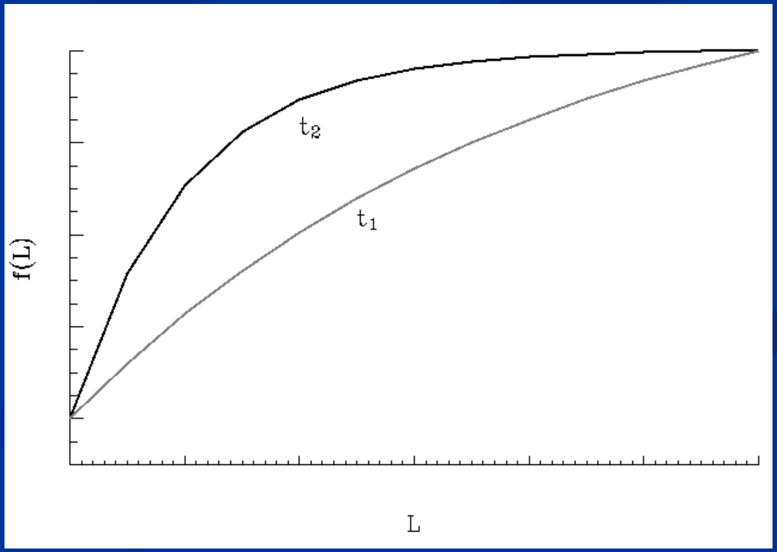


Transport in *L*: radial transport









S. Elkington, GEM 2003

Adiabatic Scaling of GOES electrons

Jenn Gannon (USGS), Terry Onsager (SWPC), Scot Elkington (LASP)

I. Convert GOES integral flux (J) to PSD at constant energy (Onsager et al.)

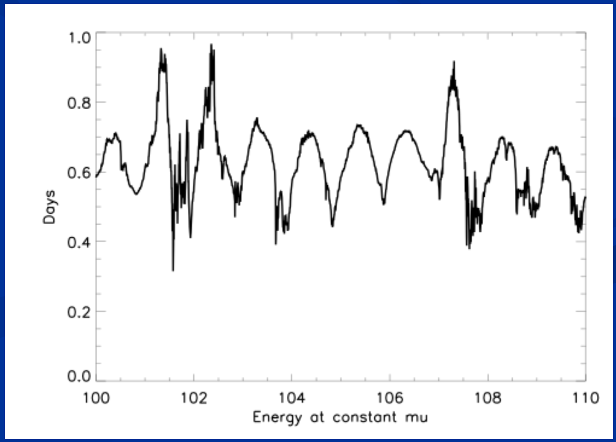
$$f(E) = \frac{c^2 J_{>E}}{(E_0^2 + EE_0)2mc^2 + 2E_0^3 + 2EE_0^2 + E^2E_0}$$

I. Estimate energy spectrum using two GOES integral flux channels

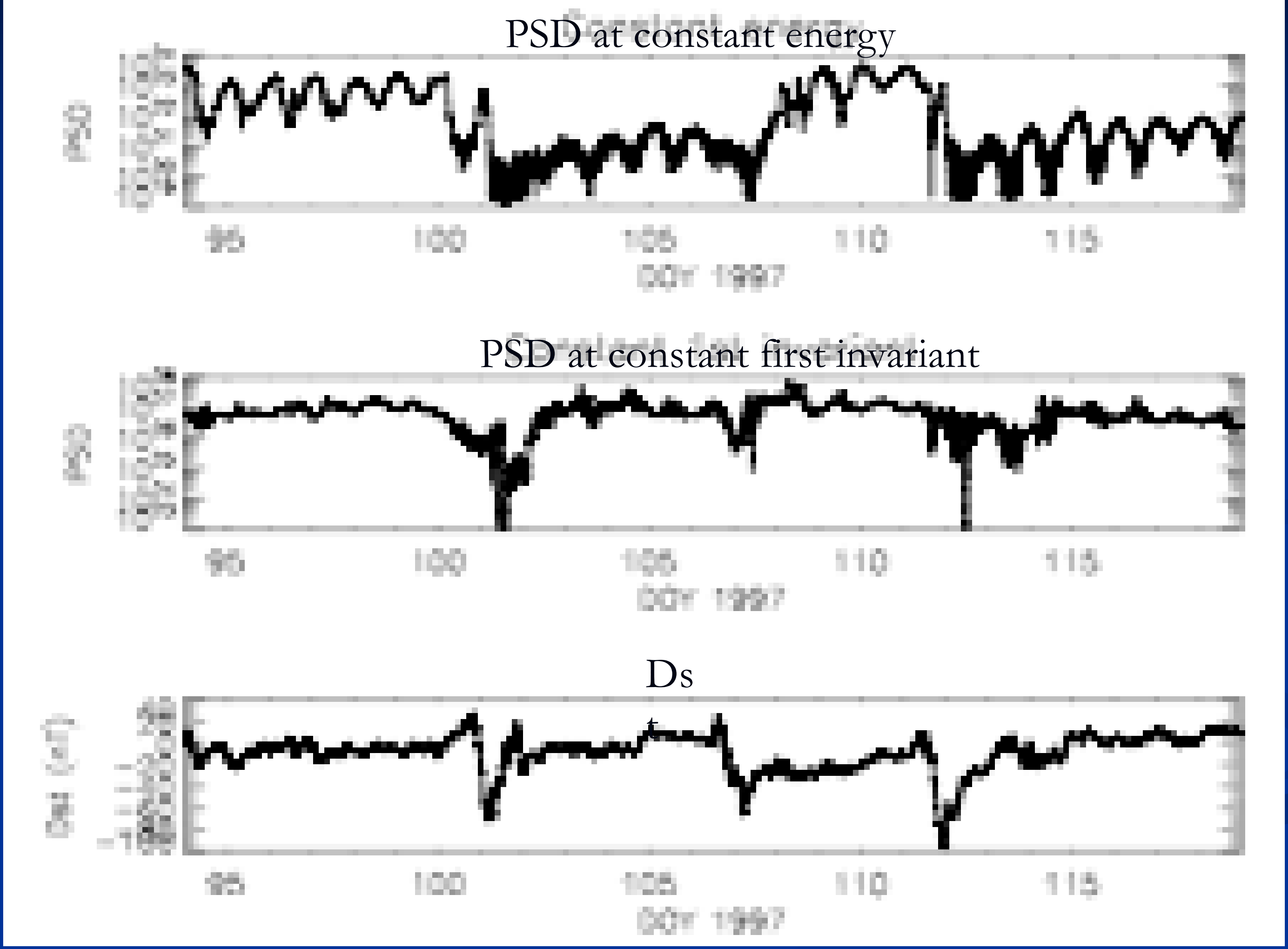
II. Scale PSD to constant *M* using estimated energy spectrum

$$f(\mu) = f(E)e^{\left(E-E(M)/_{E0}\right)}$$

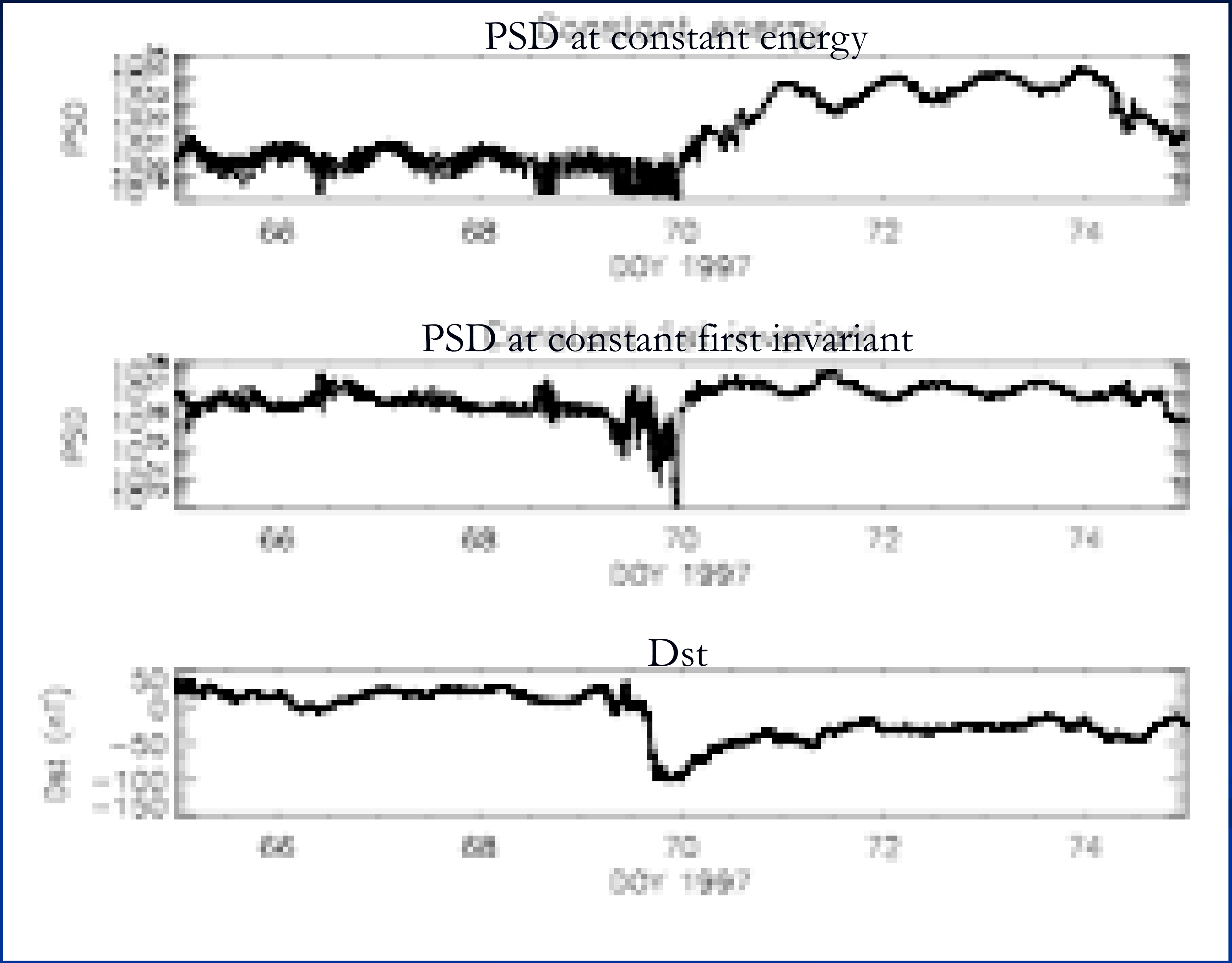
Energy at constant *M*



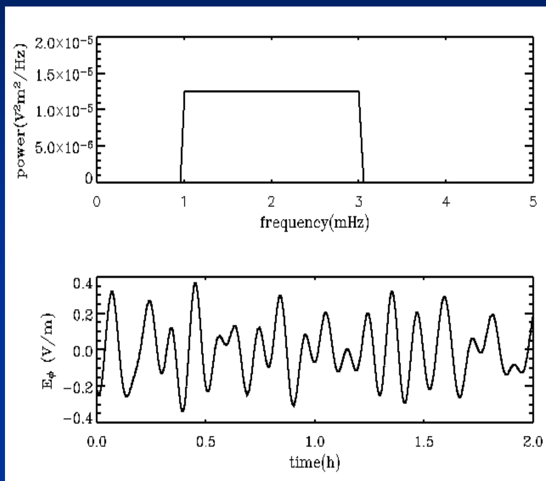
The effect of scaling



Main phase enhancement example



Quantifying radial diffusion: test particle simulations informing transport simulations



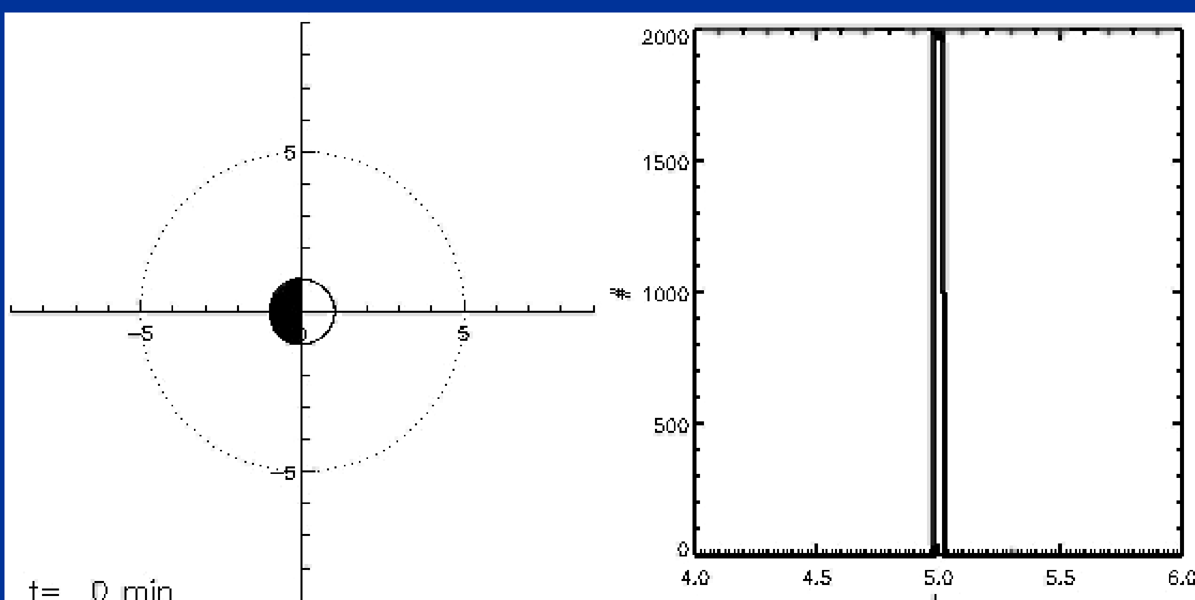
Scenario:

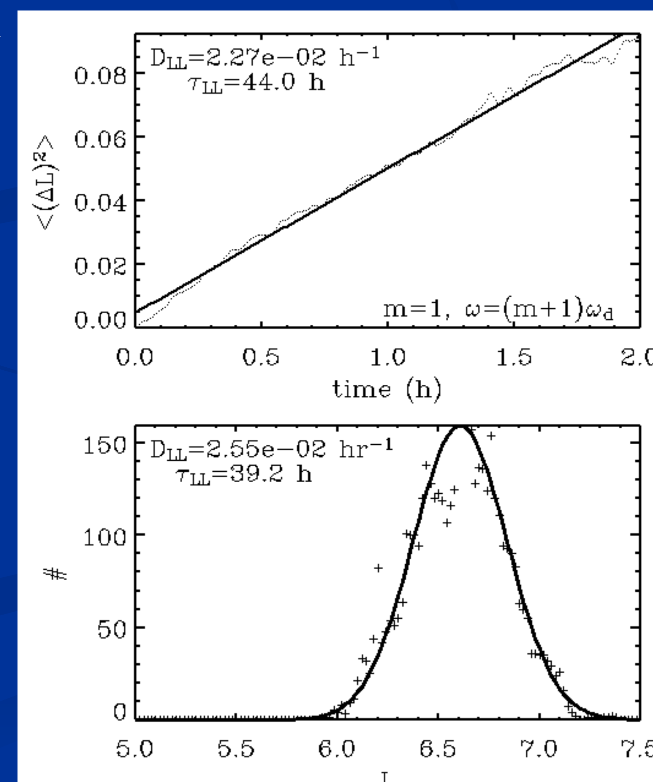
•Ensemble of particles initially at L=5 in a dipole field.

•Dynamic waves: analytic ULF with frequencies ~f_d and random phases induce radial diffusion.

Quantifying diffusion:

$$D_{LL} = \frac{\langle (\Delta L)^2 \rangle}{2\tau}$$

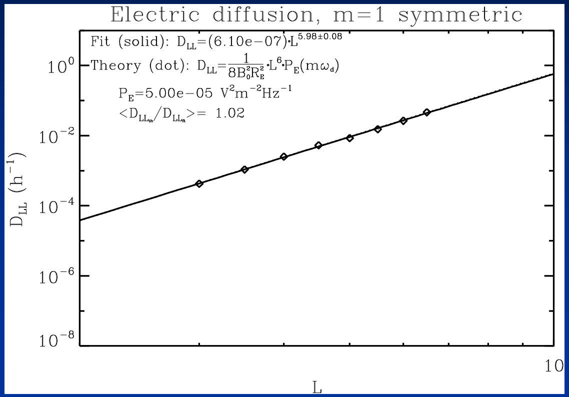




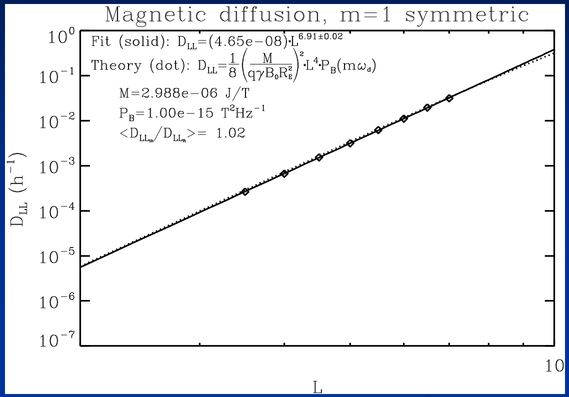
Quantifying Radial Diffusion

Different kinds of disturbances (e.g. electric vs. magnetic) and different resonant interactions (e.g. symmetric vs. asymmetric) will lead to different functional forms for the transport coefficients, D_{LL}.

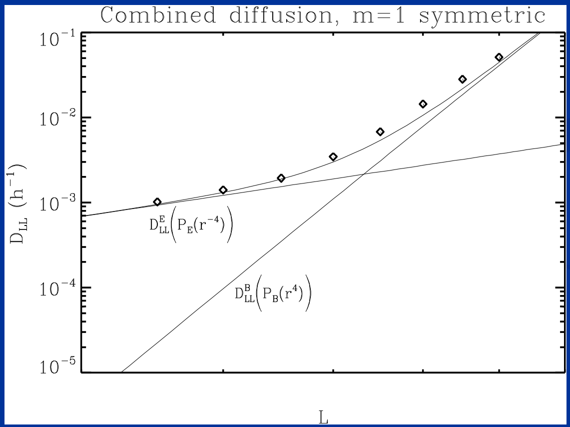
We use test particle simulations to study the rates and characteristics that result from interaction with these waves.



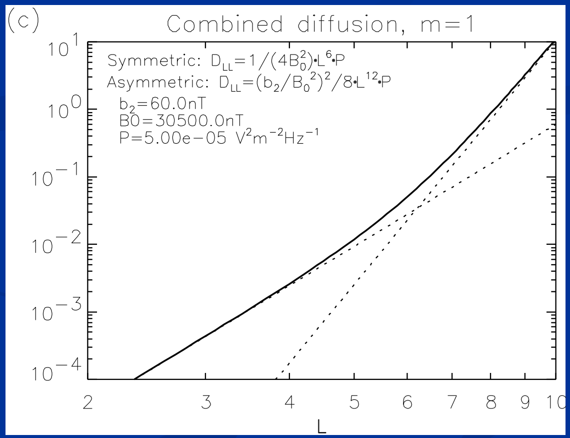
‘Electric’ diffusion



‘Magnetic’ diffusion



Electric + Magnetic

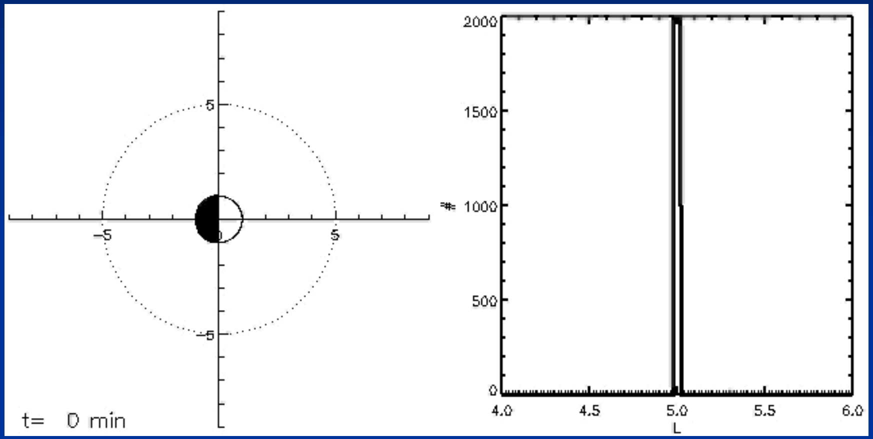


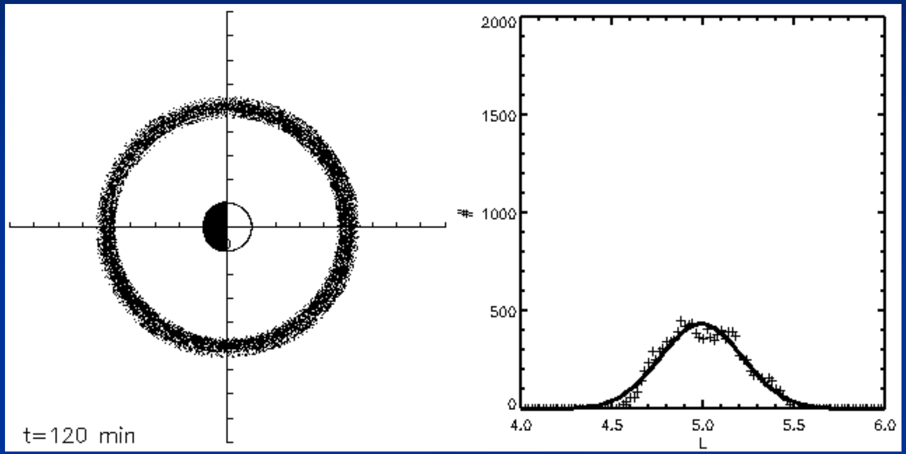
Symmetric + Asymmetric

$$D_{LL}^{tot}=\sum_m D_{LL}^{B,sym}+D_{LL}^{B,asym}+D_{LL}^{E,sym}+D_{LL}^{E,asym}$$

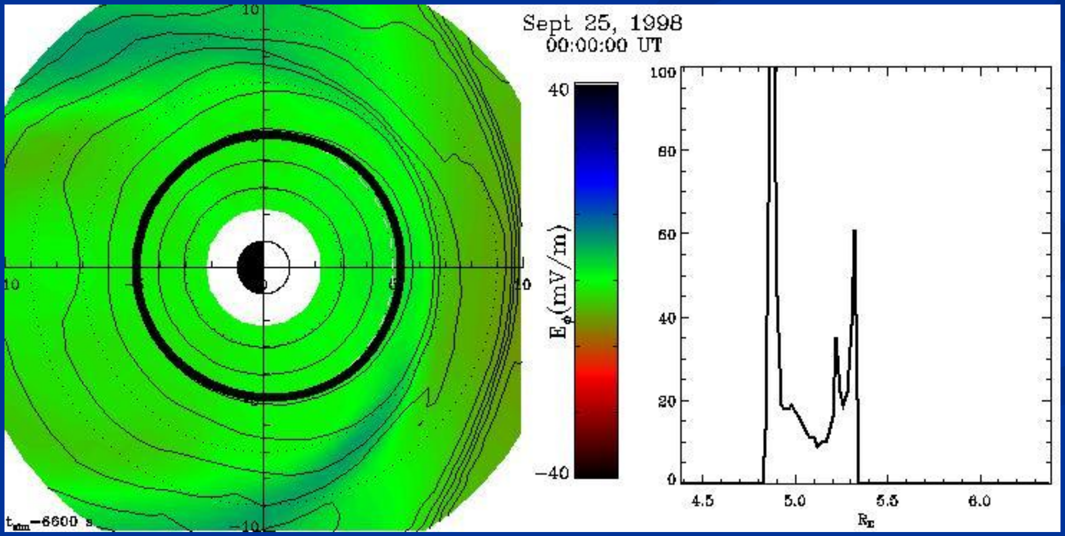
'Diffusion' of particles in an MHD simulation

We want to do something like this…





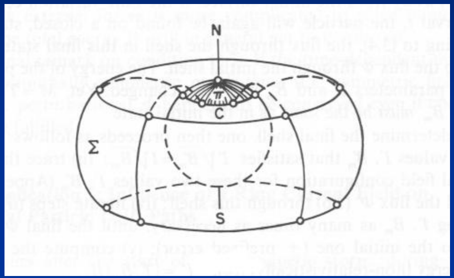
What happens if you assume $L^* \sim r/R_E$...

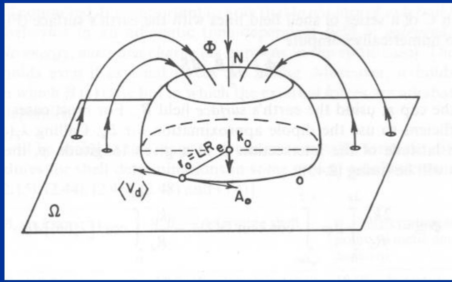


L* calculation in the MHD

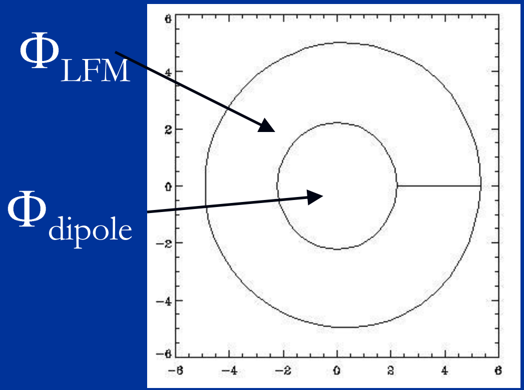
'Proper' L* calculation

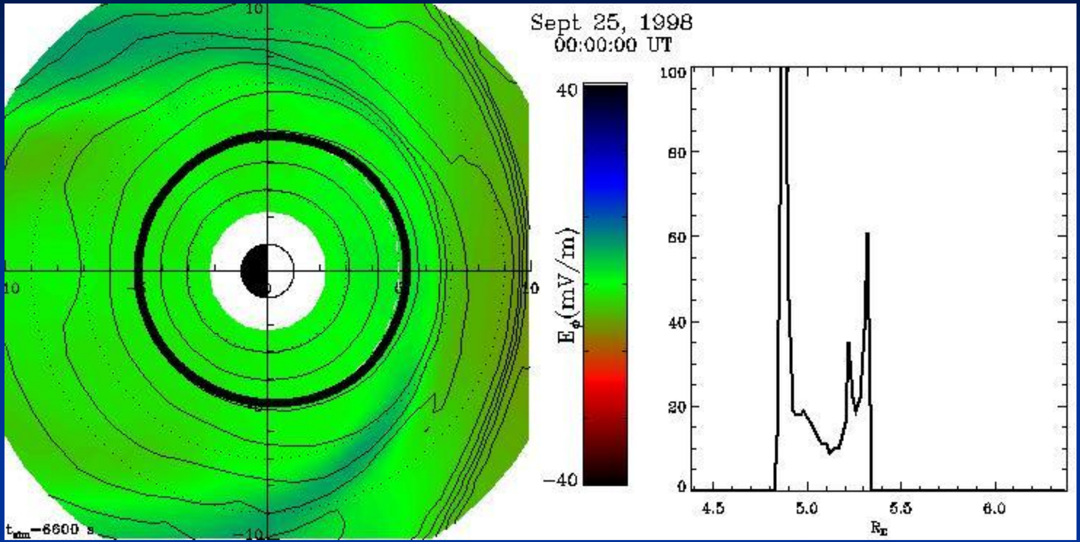
$$L^* = -\frac{2\pi B_0}{\Phi}$$



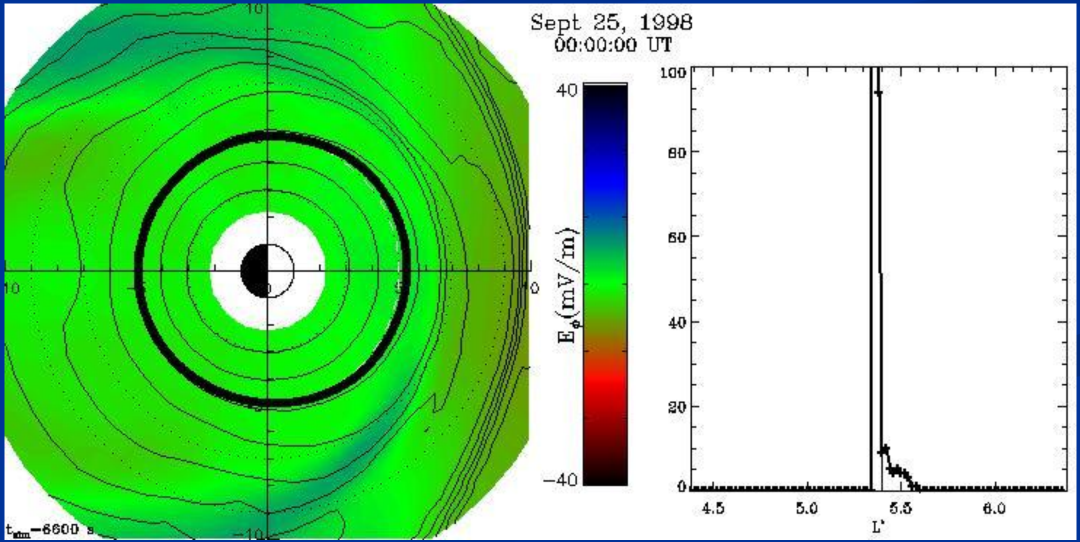


How we fudged it…





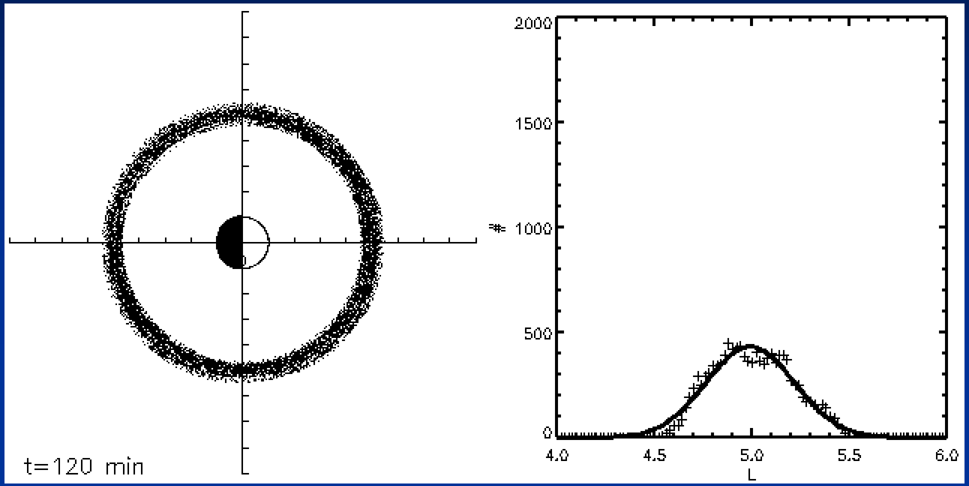
L~r/R_E



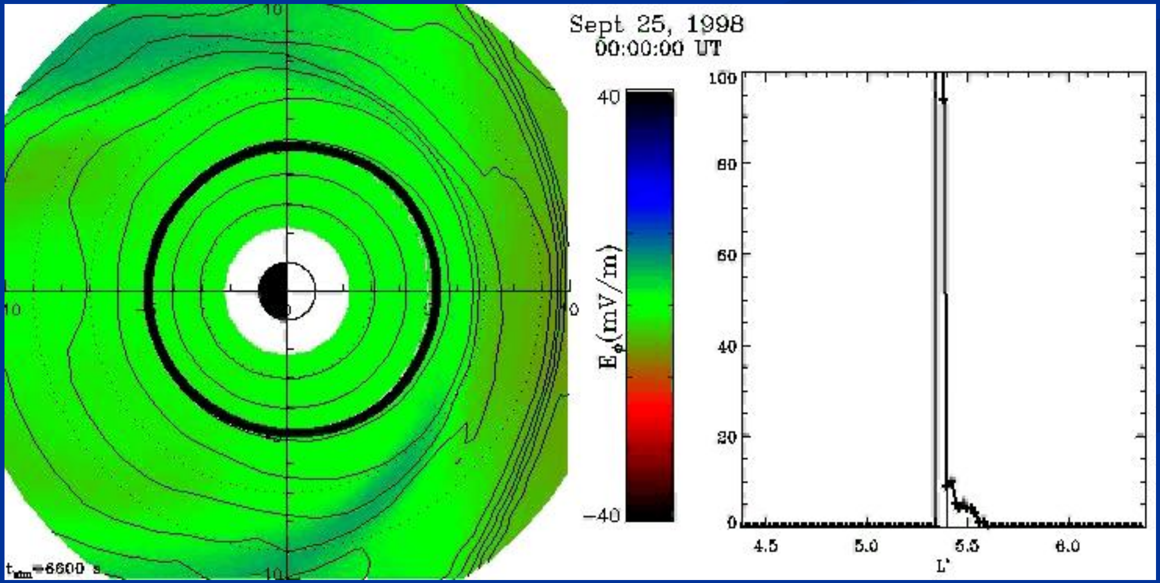
L*

‘Diffusion’ in the MHD

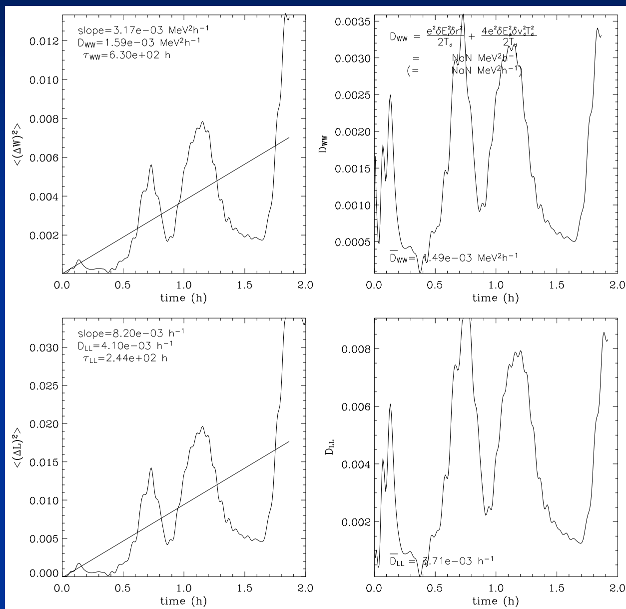
We expect:



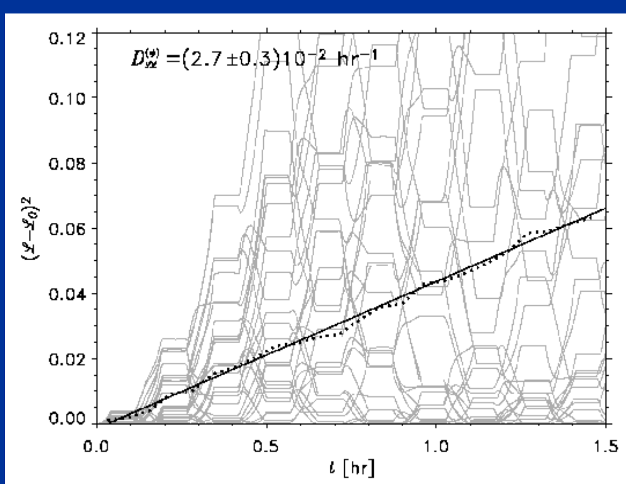
We get:



Diffusion may only be a viable description of radial transport in an 'average' sense…



Elkington et al.



Ukhorsky et al.

Comparison of Diffusion and Particle Drift Descriptions of Radial Transport in the Earth's Inner Magnetosphere

PETE RILEY AND R. A. WOLF

Department of Space Physics and Astronomy, Rice University, Houston, Texas

A comparison is made between two approaches, radial diffusion and guiding-center drift, for describing the radial motion of charged particles in the equatorial plane of the Earth's magnetosphere. For the storm event of August 1990, a time-dependent, observation-based model electrostatic field is computed. This field is then used (1) to calculate the drift of a ring of monoenergetic test particles and (2) to compute the specific radial diffusion coefficient and solve the relevant diffusion equation. Because of the electric field model employed and the data used to construct it, the particles considered were restricted to those with drift periods greater than 2 hours and therefore energies less than about 130 keV at L=3. Thus our calculations bear directly on only the low-energy part of the radiation belts. Density profiles computed from diffusion theory are compared with the results of the guiding-center simulation for a number of initial conditions. Mediocre agreement is found when the diffusion results are compared with the guiding-center simulation for the August 1990 event. However, for cases where (1) a number of electric fields are recovered from the power spectrum and the particle drifts are averaged or (2) a number of electric fields are applied sequentially to a single distribution, the agreement is considerably better. The main conclusion from these tests is that the diffusion formalism gives only roughly right answers for a single real storm but does much better for an average over a statistical ensemble of storms. Finally, several previously derived diffusion coefficients are compared with the present one as functions of energy.

Riley and Wolf (*JGR, 1994*): "The main conclusion from these tests is that the diffusive formalism gives only roughly right answers for a single real storm, but does much better on average over a statistical ensemble of storms."

parking lot

Conclusions/Summary

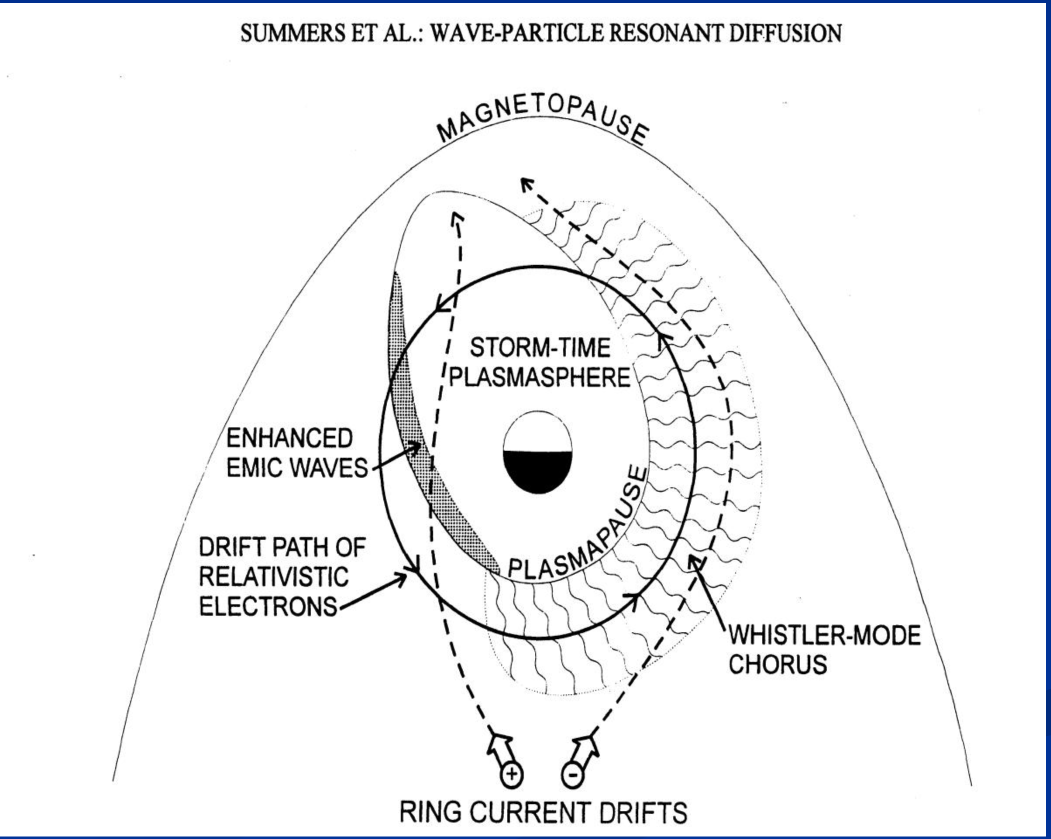
Our understanding of radial transport in the radiation belts is better, but still will require considerable effort.

- Transport equations: DLL proportionalities, energy dependence.
- ULF waves:
  - Power spectrum and occurrence characteristics of ULF waves.
  - Mode structure of magnetospheric waves.
  - How is power coupled from solar wind to inner magnetosphere?
- Boundary conditions: the plasma sheet?
  - Determine plasmasheet access.
  - Distribution in plasmasheet.

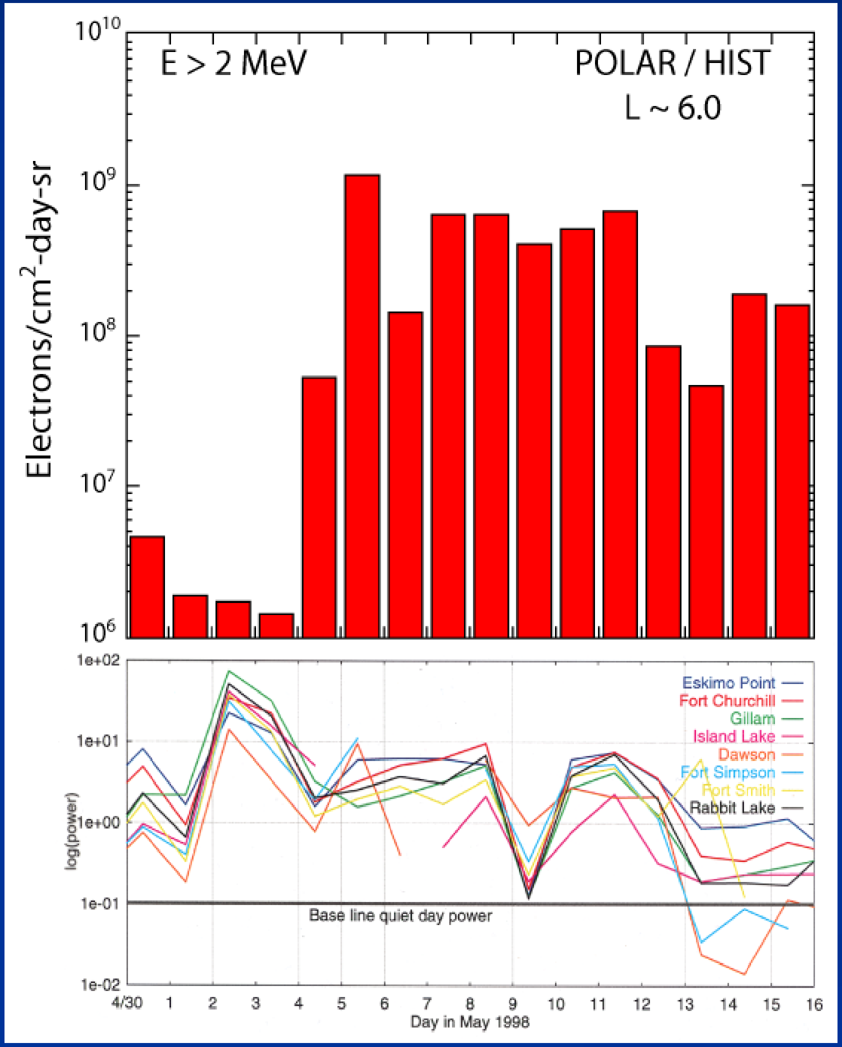
Local heating example: resonant interactions with VLF waves

Summers et al. (*JGR 103*, 20487, 1998) proposed that resonant interactions with VLF waves could heat particles:

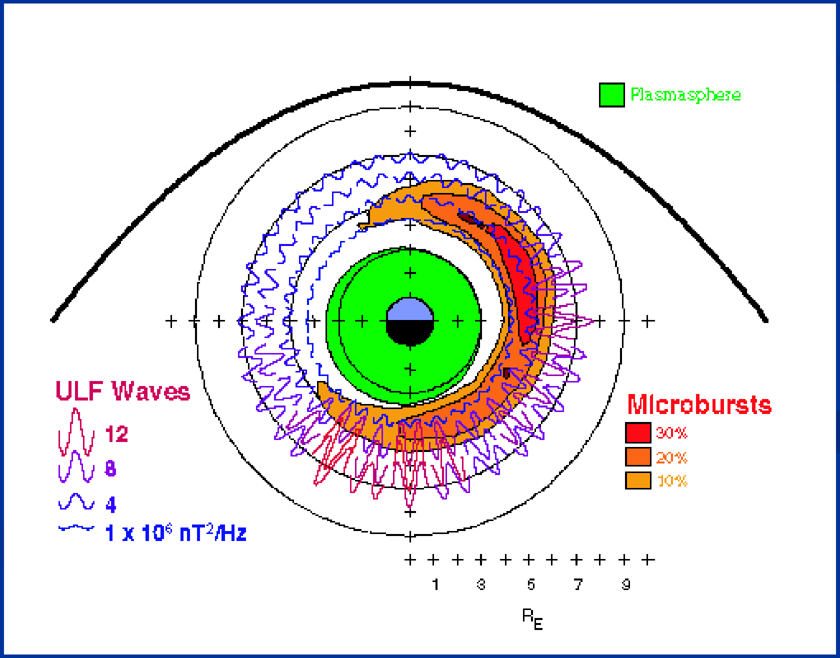
- Whistler mode chorus at dawn combined with EMIC interactions heat and isotropize particles.
- Leads to transport in M, K, and L.



Observed associations between ULF waves and radiation belt activity?



- Baker et al., *GRL 25*, 2975, 1998
- Rostoker, *GRL 25*, 3701, 1998
- Mathie & Mann, *GRL 27*, 3621, 2000
- O'brien et al., *JGR, 2003*, in press.



S. Elkington, GEM 2003