The Demonstrations & Science Experiment (DSX)

Update for RBSP Science Working Group

23-24 May 2011

Dave Lauben, Stanford

Gregory Ginet, MIT/LL

Michael Starks, AFRL

Mark Scherbarth, AFRL

In Memory…

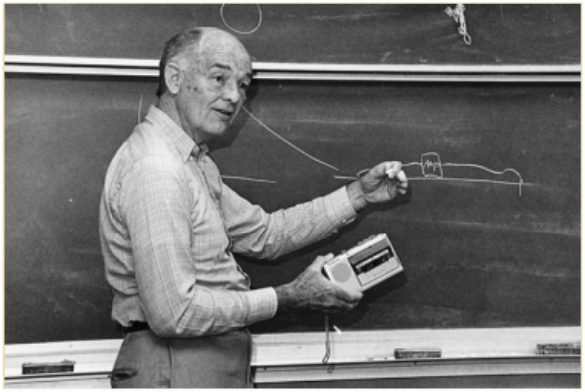
Stanford Report, May 20, 2011

Robert Helliwell, radioscience and magnetosphere expert, dead at 90

Robert Helliwell pioneered the study of how radio waves – both those naturally generated by lightning and manmade signals from a radio transmitter in Antarctica – interact with charged particles in the upper atmosphere.

BY MELISSAE FELLET

Stanford News Service



Robert Helliwell, an emeritus professor of electrical engineering, was remembered as 'a man of great ideas and inventiveness.'

Late one night in 1950, a graduate student was monitoring radio waves emitted by distant lightning when strange descending whistling tones came from a speaker. The student, Jack Mallinckrodt, mentioned the experience to his adviser, Robert Helliwell.

"I suggested that if he took a short vacation perhaps the sounds would go away," Helliwell wrote in an article for the October 1982 issue of *Stanford Engineer*. "But he didn't and they didn't. My curiosity was finally aroused and I spent a late night with Jack at the receiving station. Luckily, we both heard two distinct whistlers and I was instantly converted to belief in the reality of a strange new phenomenon."

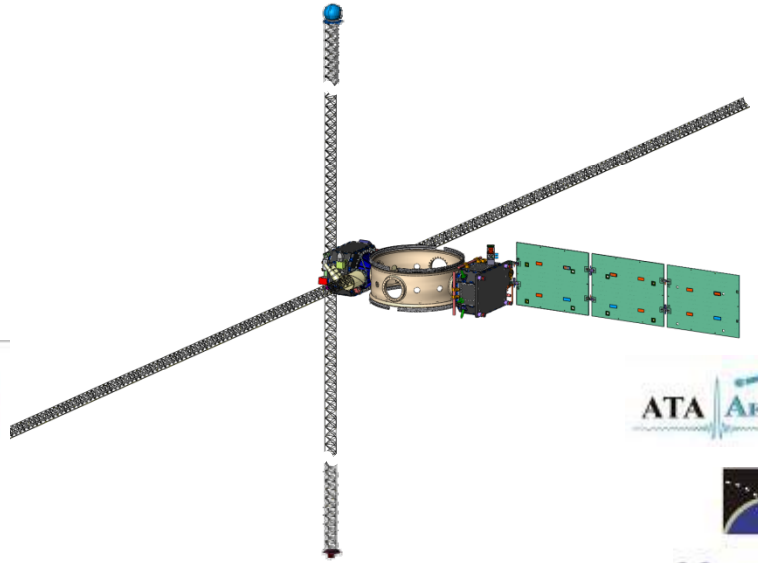
This chance observation started Helliwell on decades of research that led him from Stanford to Antarctica as he followed these mysterious radio noises and later sought to reproduce them with a transmitter. Through this work, he made fundamental discoveries about how radio waves can be used to investigate the ionized atmosphere high above Earth's surface.

"He was a pioneer," said Don Carpenter, professor emeritus of electrical engineering and one of Helliwell's colleagues. "He did some of the earliest observations and interpretations of phenomena in our field."

→ Helliwell died on May 3 in Palo Alto of complications from dementia. He was 90. A service will be held at Stanford Memorial Church on June 7 at 3 p.m.

The Team

Program Office
Systems Engineering
Integration and Test



Launch Segment

Spacecraft Bus

VLF Wave-Particle Interaction Experiment

Space Environmental Effects

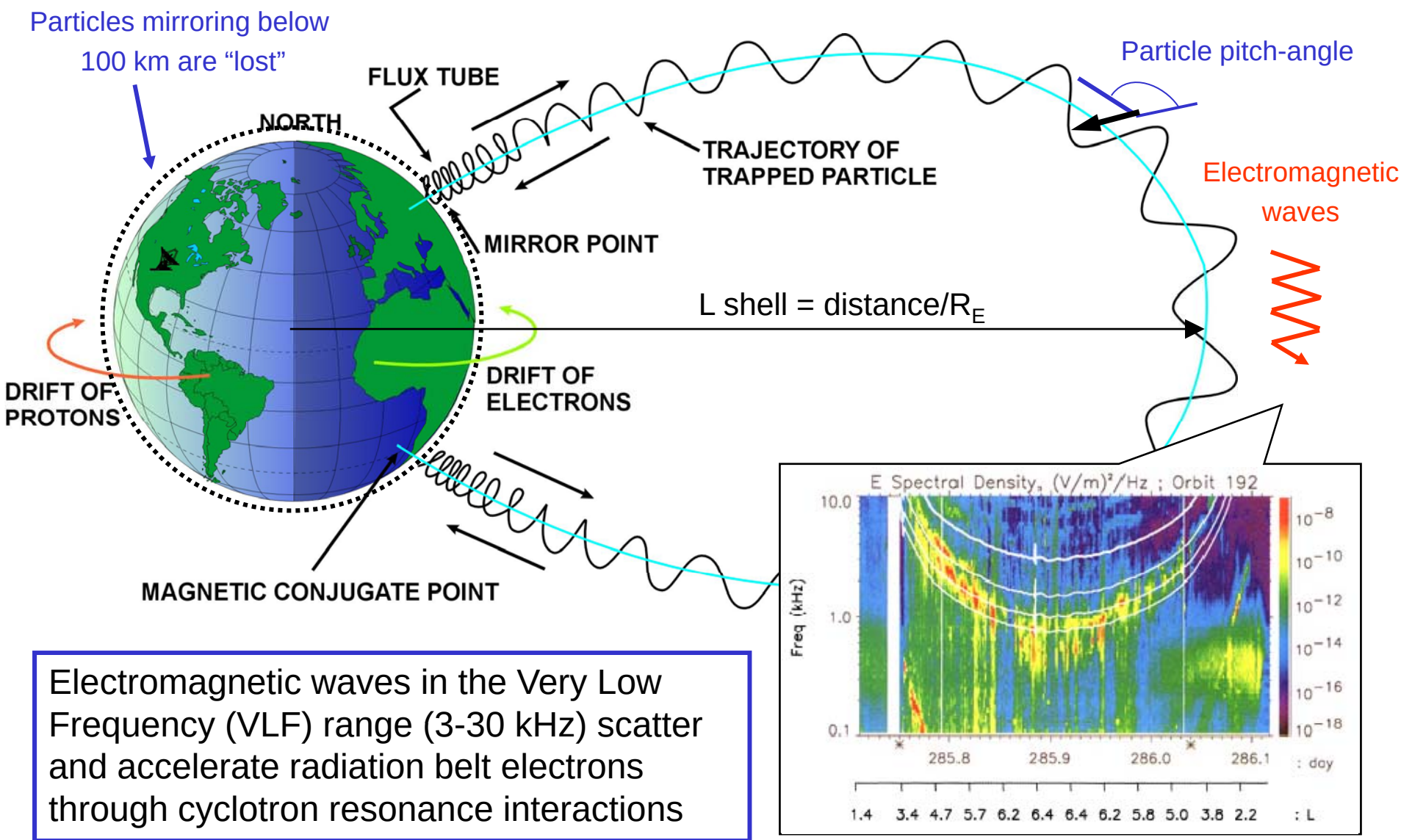
PROPULSION DIRECTORATE

Space Weather Experiments

Outline

- Introduction and Motivation
- DSX spacecraft and experimental payloads
- Science question – where is the 20 dB?
- Science question – radiation pattern in plasma?
- Conjunction opportunities
- Summary

Wave-Particle Interactions

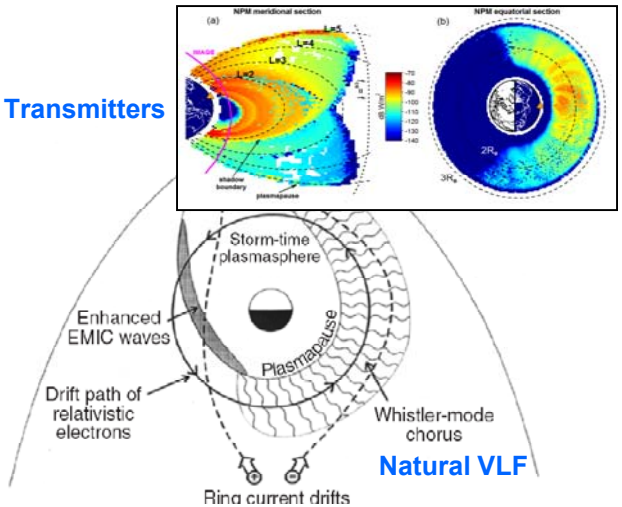


Waves from CRRES (1990)

Electromagnetic waves in the Very Low Frequency (VLF) range (3-30 kHz) scatter and accelerate radiation belt electrons through cyclotron resonance interactions

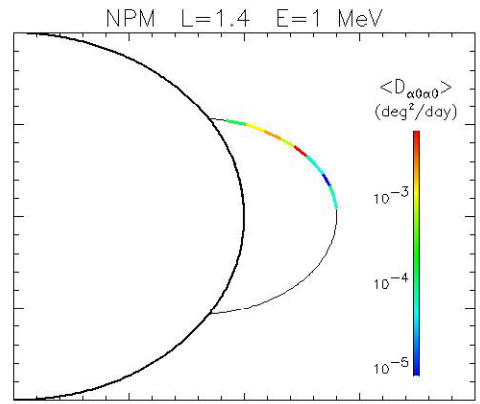
Space Weather Forecasting

Transmitters



Natural VLF

Wave power in the magnetosphere



Diffusion coefficients along field lines

$$\frac{\partial f}{\partial t} = \frac{1}{T \sin\alpha_0 \cos\alpha_0} \frac{\partial}{\partial \alpha_0}\left[D_{\alpha_0\alpha_0} T \sin\alpha_0 \cos\alpha_0 \frac{\partial f}{\partial \alpha_0}\right]$$

Particle lifetime along field lines (approximate 1D solution)

$$\frac{\partial f(\vec{X},t)}{\partial t} = \frac{1}{\Im}\sum_{ij} \frac{\partial}{\partial X_i}\left[\Im D_{X_i X_j} \frac{\partial f(\vec{X},t)}{\partial X_j}\right]$$

Full 3D global, time dependent particle distributions

X_i = (L, E, α)

$$\omega - k_{\|} v_{\|} = -n \frac{\Omega_e}{\gamma}$$

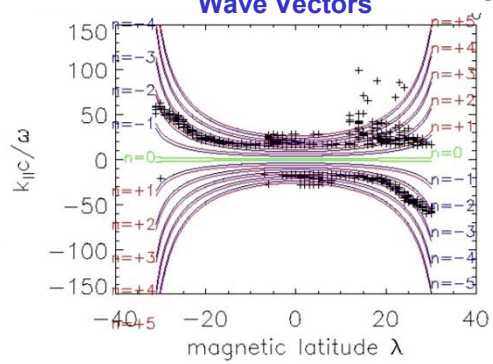
Wave-particle resonance condition

$$D_{\alpha_0\alpha_0} = \frac{\pi}{2} \sum_n \frac{\cos^7\lambda}{T\cos^2\alpha_0} \frac{\Omega_e^2}{\eta_{\|}} \left(\frac{\partial\omega}{\partial\lambda}\right)^{-1} \frac{B_w^2}{B^2} \frac{j_n \Phi_n(\theta)}{\gamma^2 (v/c)},$$

$$j_n = \begin{cases} 1, & n \neq 0, \\ \sin^4\alpha, & n = 0 \end{cases}$$

Diffusion coefficients = sum over resonances

Distribution of Resonant Wave Vectors

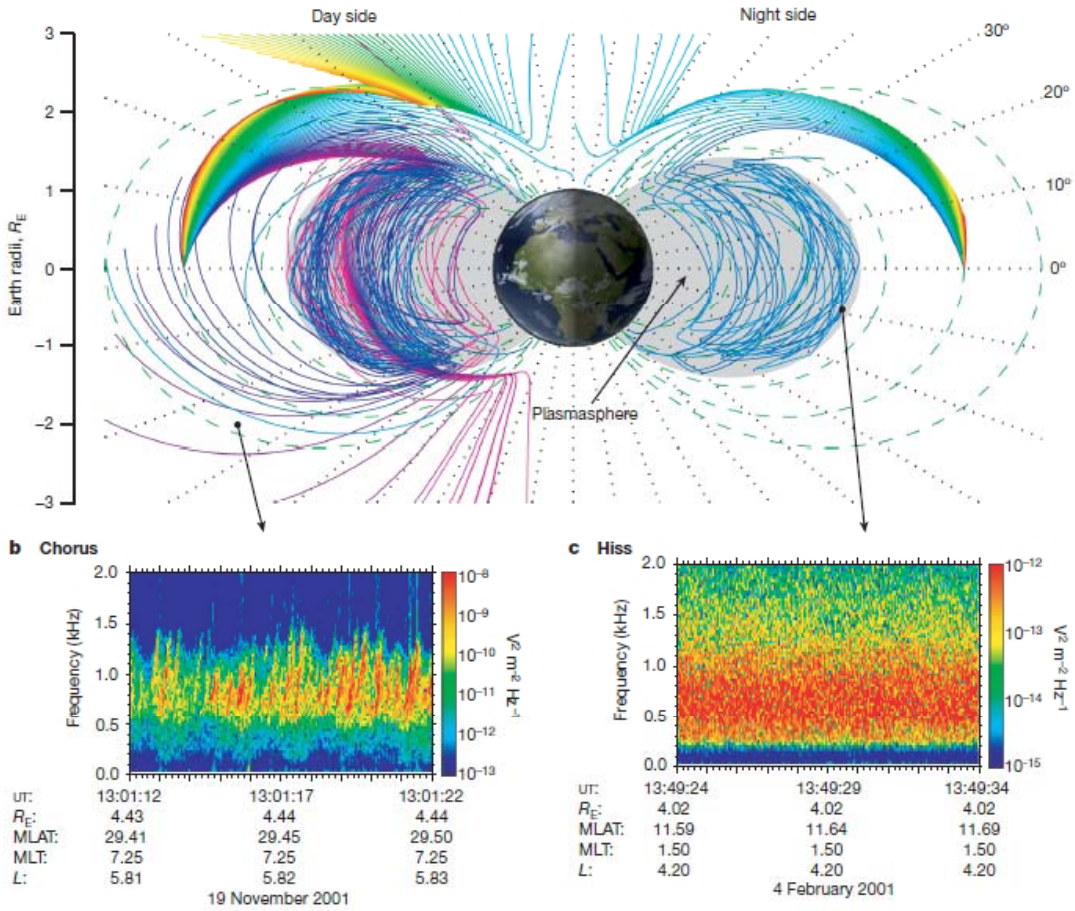


Complex dependence on energy, frequency, and pitch angle

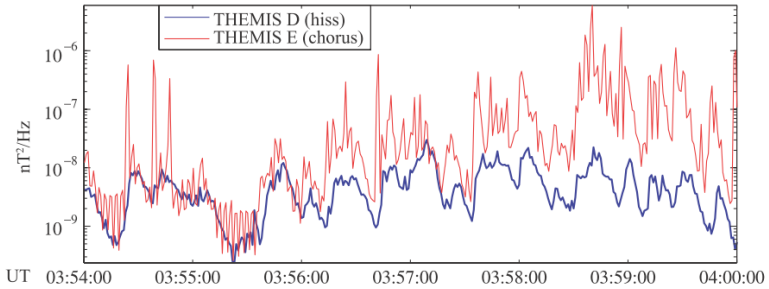
Quantitative understanding of VLF wave power distribution & resultant wave-particle interactions is crucial for radiation belt specification & forecasting

Non-Local Origins of Hiss

[Bortnik et al., Nature 452, 6 March 2008]



[Bortnik et al., Science 324, 8 May 2008]



- **Whistler-mode hiss helps control the lifetime of radiation belt electrons.**
- **Hiss may originate in chorus waves, and enter the plasmasphere via high latitude ray paths**

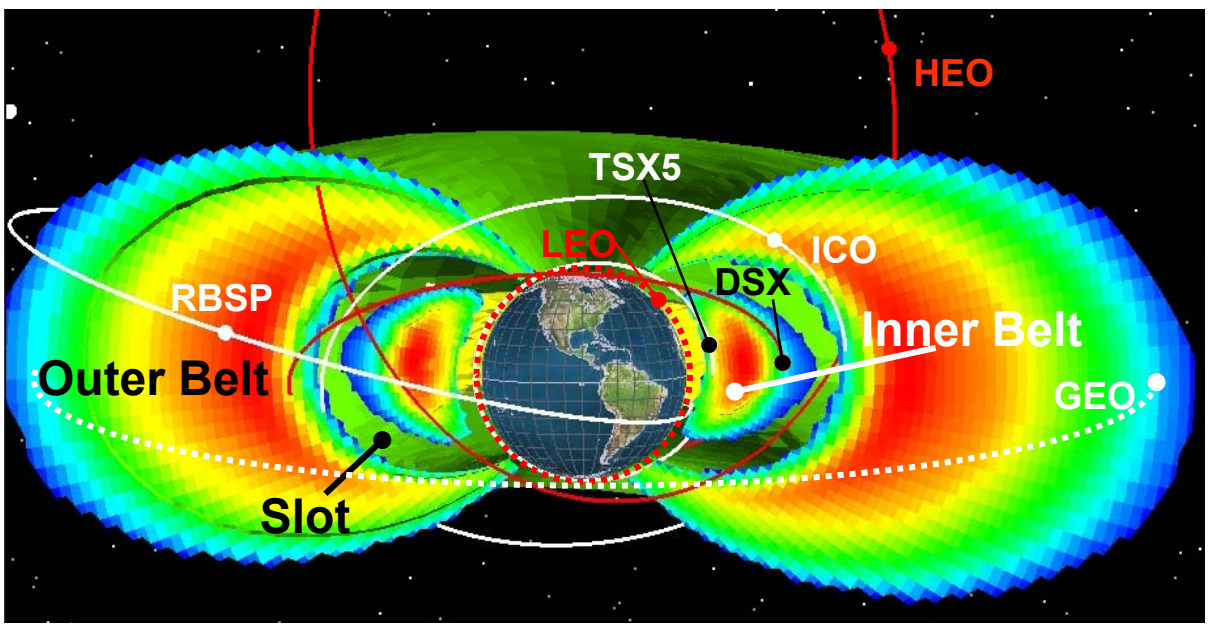
Conjunction experiments with space-based VLF transmitters and receivers can definitively probe the nature of ray-paths.

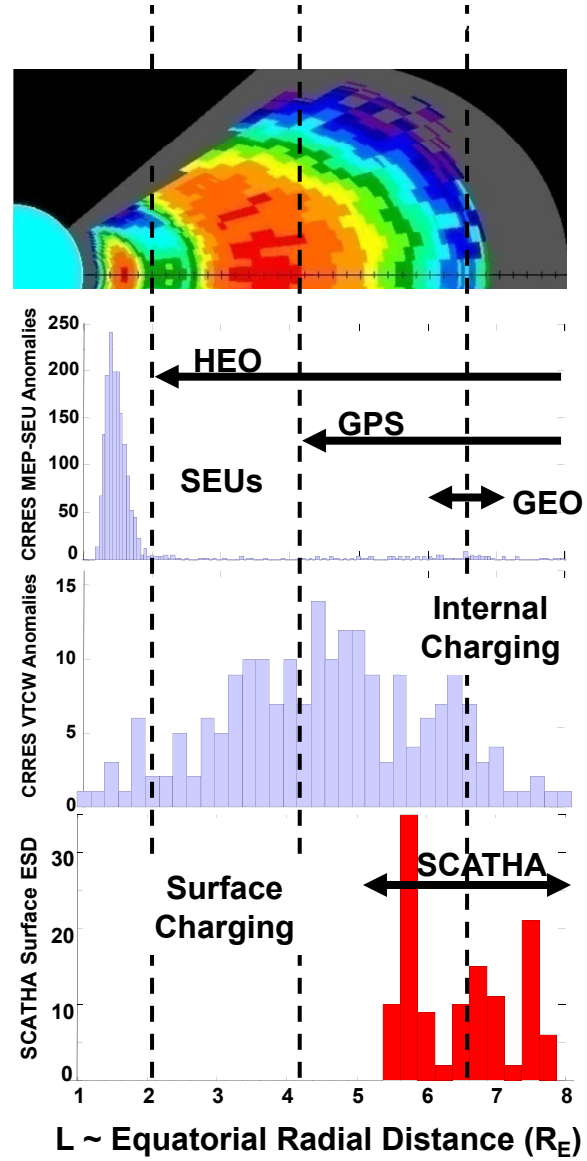
Map the MEO Environment

Satellite designers need a definitive model of the trapped energetic particle and plasma environment to include:

- Quantitative accuracy
- Indications of uncertainty
- Flux probability of occurrence and worst cases for different exposure periods
- Broad energy ranges
- Complete spatial coverage

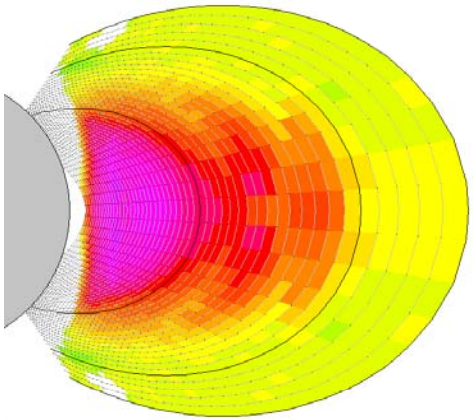
MEO is sorely under sampled!





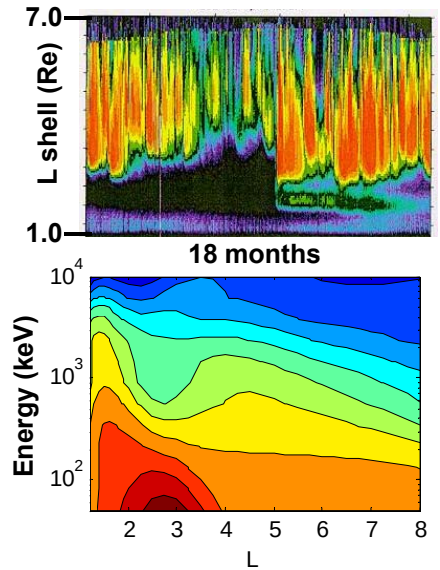
New Radiation Belt Model AP9/AE9

Satellite data



+

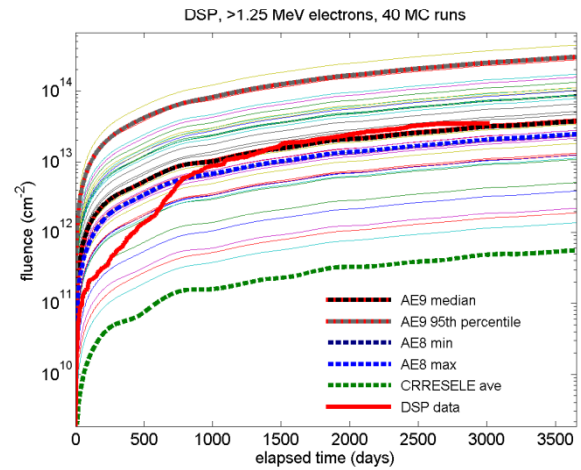
Statistical & physics based analysis



Statistical Monte-Carlo Model

=

Mission orbit



User application

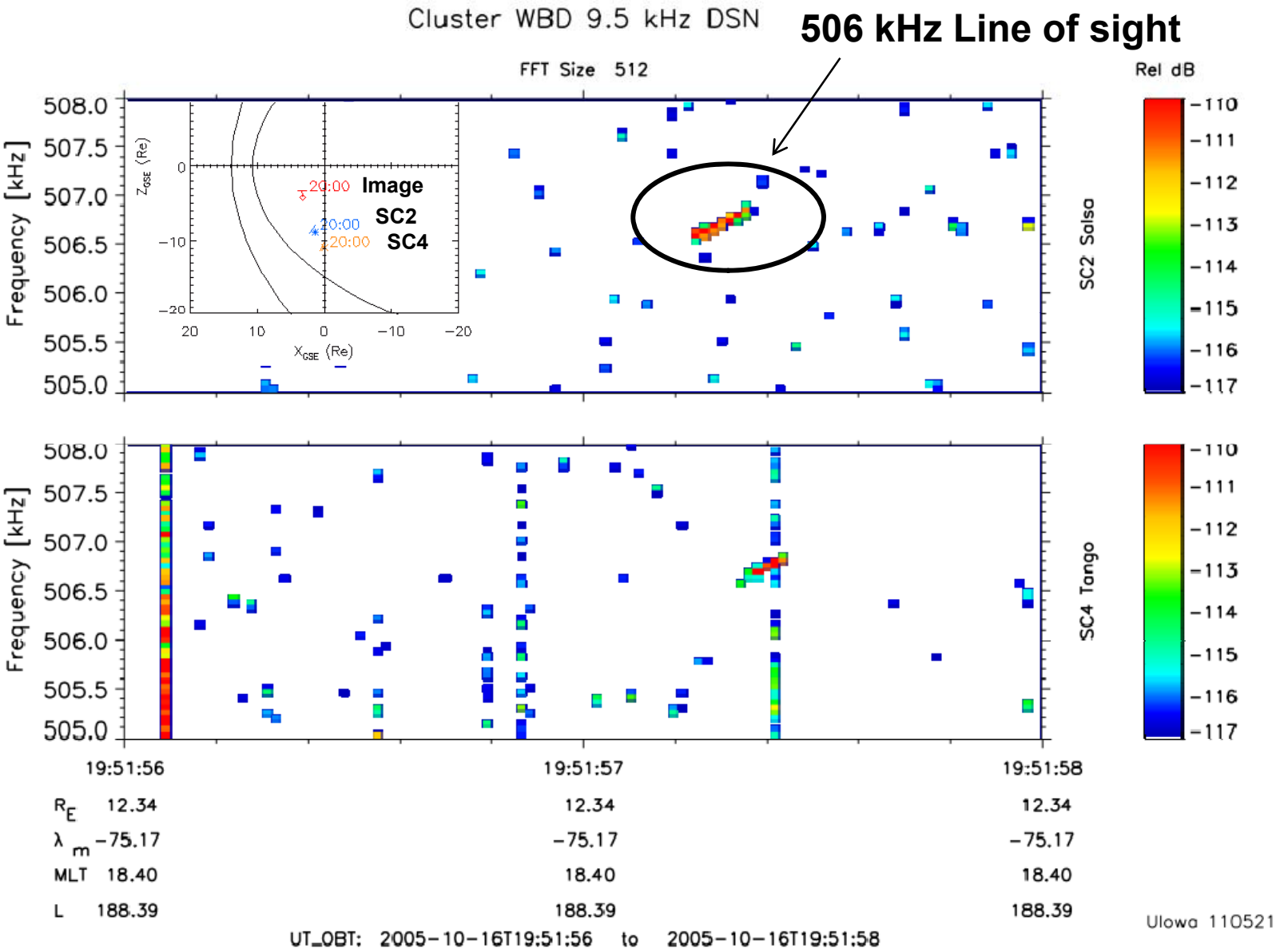
- New AP-9/AE-9 model being developed by NRO - AFRL - Aerospace – MIT/LL - LANL consortium
- Provides significant improvement in spectral coverage, error estimation and statistical output
- Needed by satellite engineers to control risk, maximize capability and reduce cost
- Version Beta released Apr 2010 and now being evaluated by 20+ independent spacecraft engineers from industry and government – Version 1.0 due in June 2011
- Version 2.0 (~2015) will utilize measurements from NASA Radiation Belt Storm Probes (RBSP) and AFRL DSX missions

Past Active VLF Space Experiments

Satellite	Orbit	Epoch	TX (RX) Freq. [kHz]	TX Type	Comments
LOFTI I	166 x 960 km 28.4°	21 Feb 61 – 30 Mar 61	(18)	-	First space VLF experiment designed to measure ground stations (NBA , Panama).
LOFTI - IIA	170 x 925 km 70°	15 Jun 63 – 15 Jul 63	(10-18)	-	Focus was on impedance variations as functions of environment.
OV3-3	362 x 4488 km 81.6°	4 Aug-66 – 30 May 67	Broadband	Solar cell current noise	Focus on plasma density influences on impedance variation.
OV1-21S (1971-67A)	800 x 921 km 87.6°	7 Aug 71 – 3 Sep 71	0.4 – 14.5	Electric dipole: 2 x 16 m elements, Voltage: 10 mV - 100 V peak-to-peak	Observed impedances are in reasonable agreement with linear plasma sheath dominated predictions. No far-field measurements.
OV1-20S (1971-67B)	75 x 1948 km 92°	7 Aug 71 – 28 Aug 71	300	Electric dipole: 2 x 1.27 cm elements, 2.45 cm separation Voltage: 30 V	Observed resonance cone and angle/width compared favorably to cold and warm plasma theory predictions.
Activny	500 x 2500 km 82.6°	28 Sep 88 – 30 Apr 90	9.5 – 10	Magnetic loop: 20 m diameter, 1 m thick	Antenna did not deploy properly. Measurements by DE-1 indicate < 10 mW radiated power (no detections.)
*IMAGE	1000 x 45922 km 40°, precesses	25 Mar 00 - 18 Dec 05	3 - 3000	Electric dipole: X: 125 m - 250 m Power: 0.1 mW -10 W	Not optimized for VLF transmission but good sheath impedance measurements.
DSX	6000 x 12000 km 120° retrograde	Oct 2012 (pending)	3 – 50000	Electric dipole: Y: 80 m tip-to-tip	

*Heritage for DSX

Image/RPI Chirp Rx by Cluster/WBD



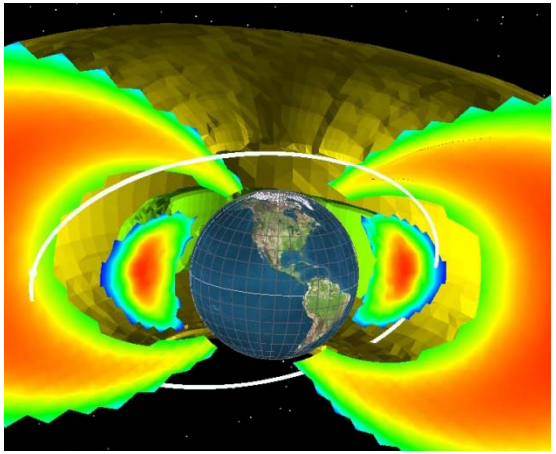
DSX Mission Objectives

Three science experiments:

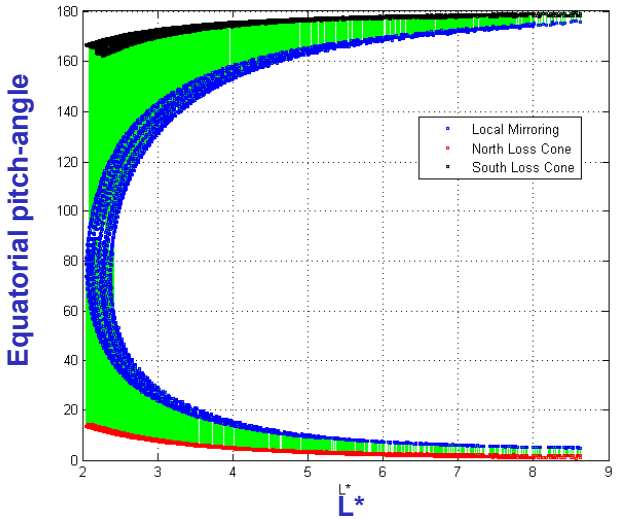
1) **Wave-particle interactions (WPIx)**
   - Determine efficiency of injecting VLF into space plasmas *in situ*
   - Determine global distribution of natural & man-made ELF-VLF waves
   - Characterize and quantify wave-particle interactions

2) **Space weather (SWx)**
   - Map MEO radiation & plasma environment
   - Diagnose in-situ environment for wave generation experiments

3) **Space environment effects (SFx)**
   - Quantify effects of MEO environment on new technologies
   - Determine physical mechanisms responsible for material breakdown



6000 x 12000 km, 120°, launch ~ Oct 2012



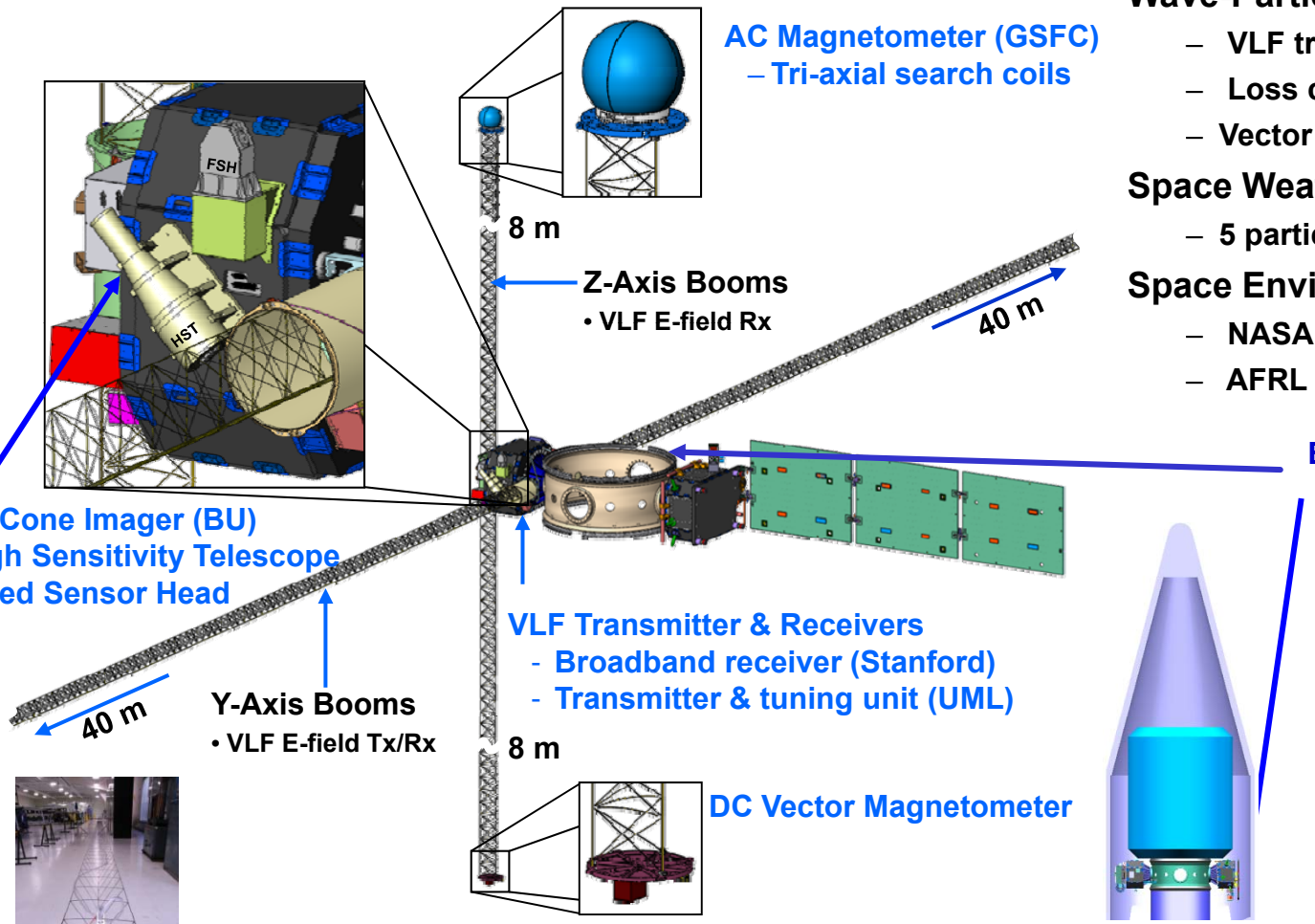
Magnetic phase space

DSX Experimental Apparatus

Wave-Particle Interactions (WPIx)
- VLF transmitter & receivers
- Loss cone imager
- Vector magnetometer

Space Weather (SWx)
- 5 particle & plasma detectors

Space Environmental Effects (SFx)
- NASA Space Environment Testbed
- AFRL effects experiment



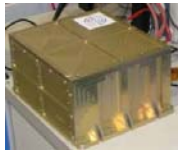
DSX satellite

Boom deployment test

DSX being integrated!

Wave-Particle Interactions Payload (WPIx)

- **Receiver** (Stanford, Lockheed-Martin, NASA/Goddard):
  - Three search coil magnetometers (3 B components)
  - Two dipole antennas (2 E components)
  - Frequency range: 100 – 50 kHz
  - Sensitivity 1.0e-16 $V^2/m^2/Hz$ (E) & 1.0e-11 nT^2/Hz (B)
- **Transmitter** (UMass Lowell, SWRI, AFRL):
  - 3 – 50 kHz at up to 5 kV (9 kV at end of life)
  - 50 – 3000 kHz at 1W (local electron density)
- **Loss Cone Imager** (Boston University, AFRL)
  - High Sensitivity Telescope (HST): measures 100 – 500 keV e- with 0.1 cm^2-str geometric factor within 6.5 deg of loss cone
  - Fixed Sensor Heads (FSH): 130 deg x 10 deg of pitch angle distribution for 50 – 700 keV electrons every 167 msec
- **Vector Magnetometer** (UCLA, UMich)
  - 0 – 8 Hz three-axis measurement at ±0.1 nT accuracy



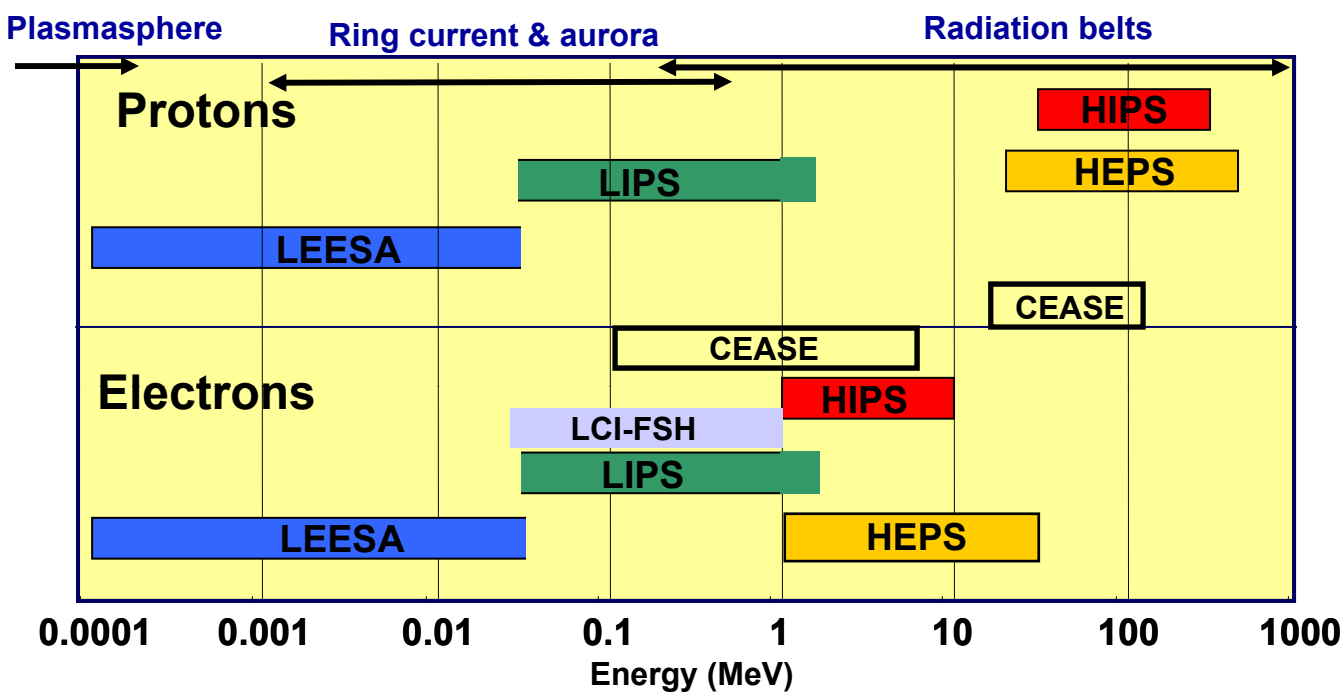

Transmitter control & tuning units

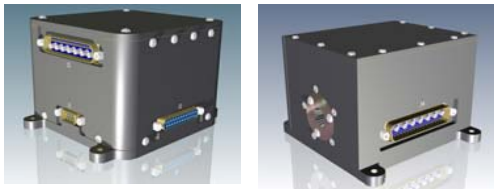

Broadband receiver & tri-axial search coils

Loss Cone Imager HST & FSH

Vector magnetometer

Space Weather Payload (SWx)

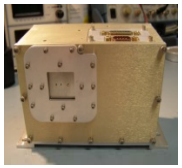




HEPS

HIPS

CEASE



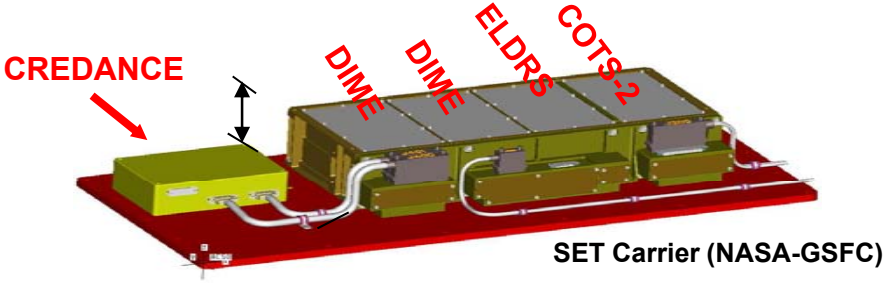
LIPS

LEESA

CEASE - Compact Environment Anomaly Sensor (Amptek, AFRL)
LEESA - Low Energy Electrostatic Analyzer (AFRL)
LIPS - Low Energy Imaging Particle Spectrometer (PSI)
HIPS - High Energy Imaging Particle Spectrometer (PSI)
HEPS - High Energy Particle Sensor (Amptek, ATC)

Comprehensive SWx sensor suite will map full range of MEO space particle hazards

Space Weather Effects Payload (SFx)



SET Carrier (NASA-GSFC)

NASA Space Environment Testbed (SET)

- Correlative Environment Monitor (QinetiQ)
  - Dosimeter & deep-dielectric charging package
- DIME (Clemson Univ)
  - Dosimetry Intercomparison and Miniaturization
- ELDRS (Arizona State)
  - Development of space-based test platform for the characterization of proton effects and Enhanced Low Dose Rate Sensitivity (ELDRS) in bipolar junction transistors
- COTS-2 (CNES and NASA)
  - Validation of single event effects mitigation via fault tolerant methodology

Photometers

1”

Radiometers

AFRL/PRS “COTS” sensors

Objective: directly measure changes in

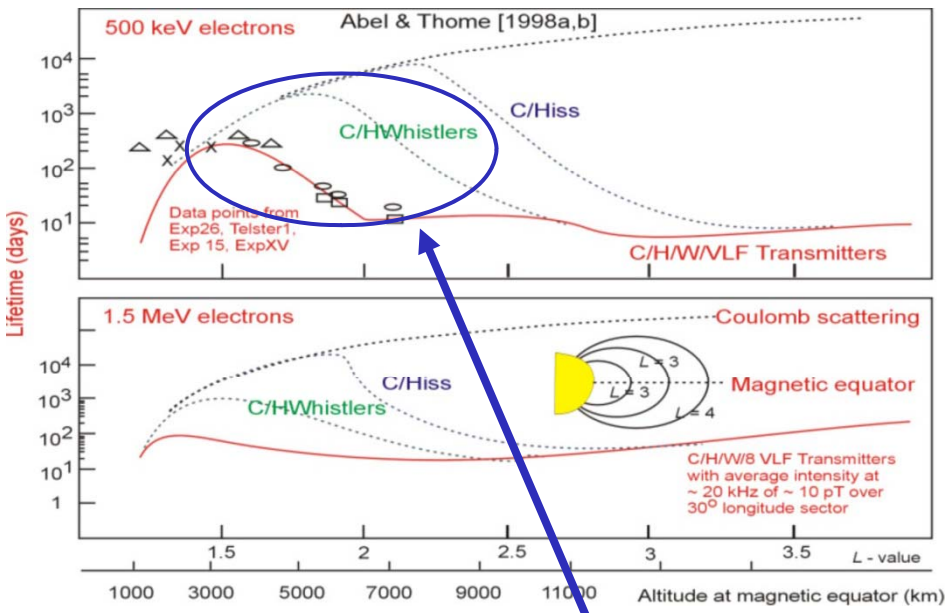
- Optical transmission,
- Thermal absorption
- Thermal emission

due to MEO radiation environment

SFx experiments will quantify MEO environment effects on advanced spacecraft technologies & determine basic physics of breakdown

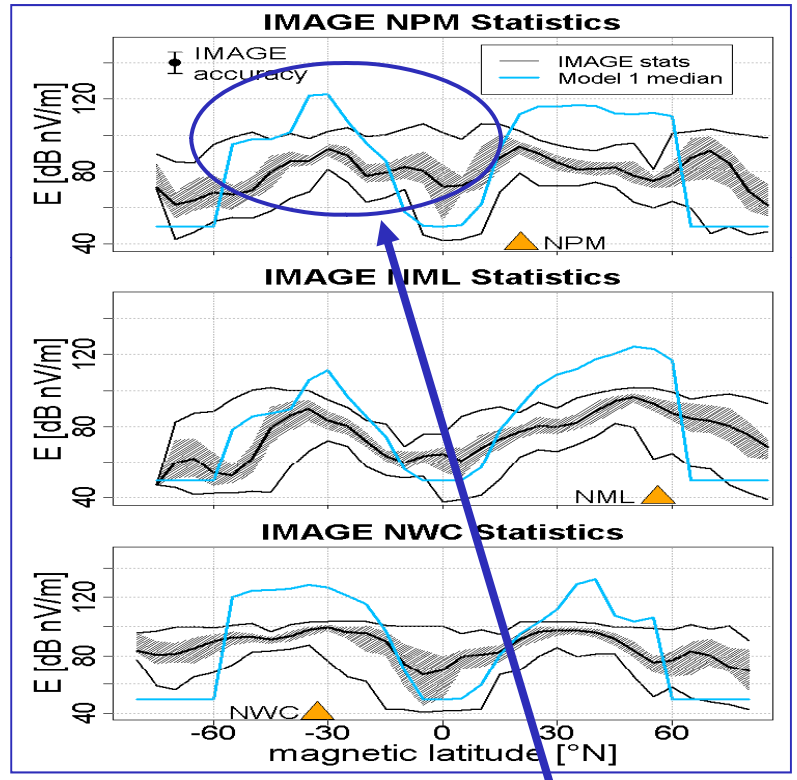
Where is the 20 dB?

Abel & Thorne (1998)



≠

Starks, et al. (2008)

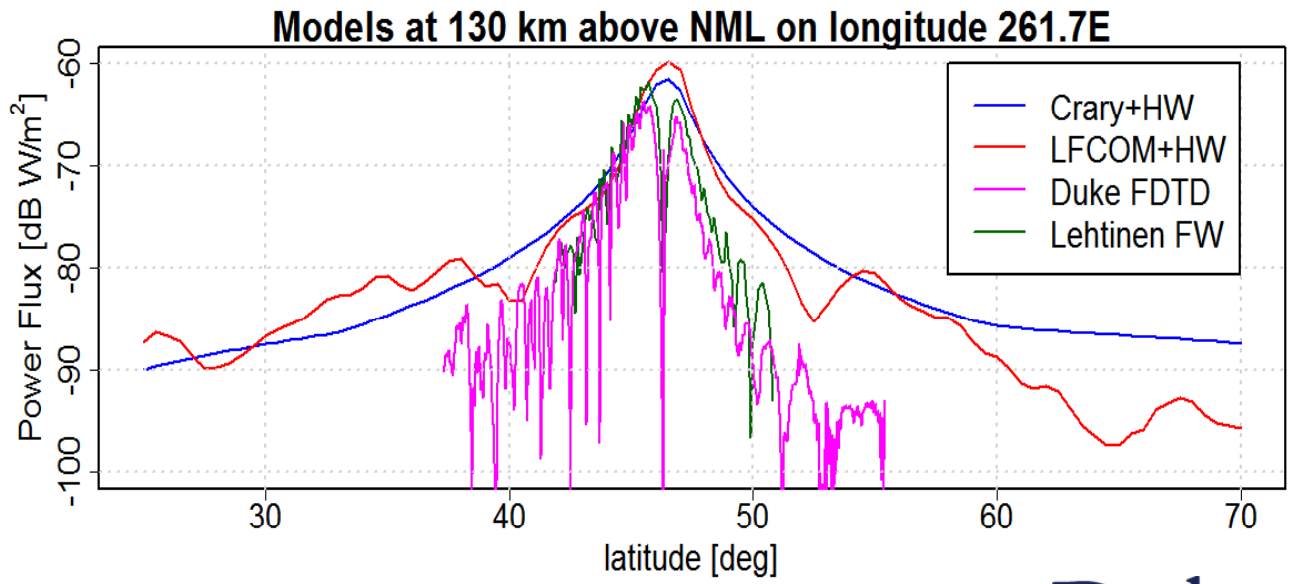


Ground transmitter VLF power needed in the inner magnetosphere… but where is it?

20dB? It's not the Absorption Model…

- The four models operate entirely differently: empirical, mode theory, finite differences, full wave

- All predict essentially the same ionospheric penetration fields



- However, all of them *overestimate the fields by 20 dB or more*.

Questions:

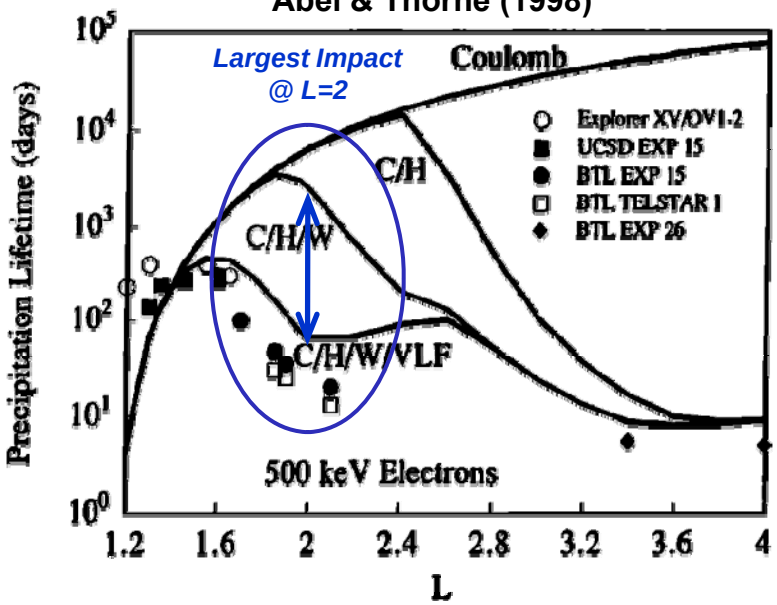
- Where is the transmitter power going?
  - Non-linear lower-hyrbid wave – density fluctuation scattering?
- What is scattering the particles at L < 2 ?
  - Could lightning be more effective then previously thought?

20db? What About Wave Type?

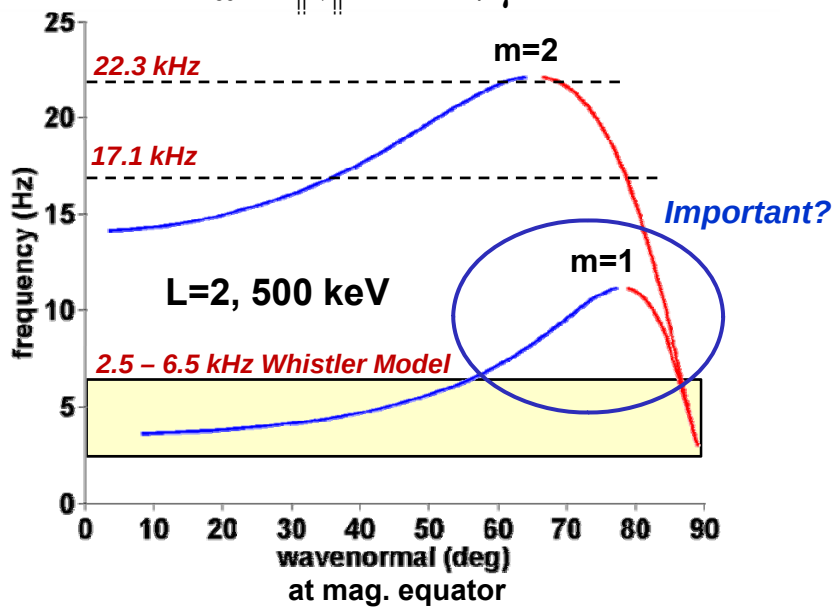
Table 1. Adopted Wave Parameters, in Abel & Thorne (1998a)

Type of Wave	Plasmaspheric Hiss	Lightning-Generated Whistlers	VLF Transmitter Signals
Frequency/bandwidth, Hz	550 ± 300 Hz *(250 – 850 Hz)*	4.5 ± 2 kHz *(2.5 – 6.5 kHz)*	17.1 kHz ± 50 and 22.3 kHz ± 50 Hz
Propagation angle/width	45°± 22.5°	45°± 22.5°	45°± 22.5°
Wave intensity	10 pT	10 pT	variable
Occurrence rate	10% to 50%	3%	2.4% each

Abel & Thorne (1998)



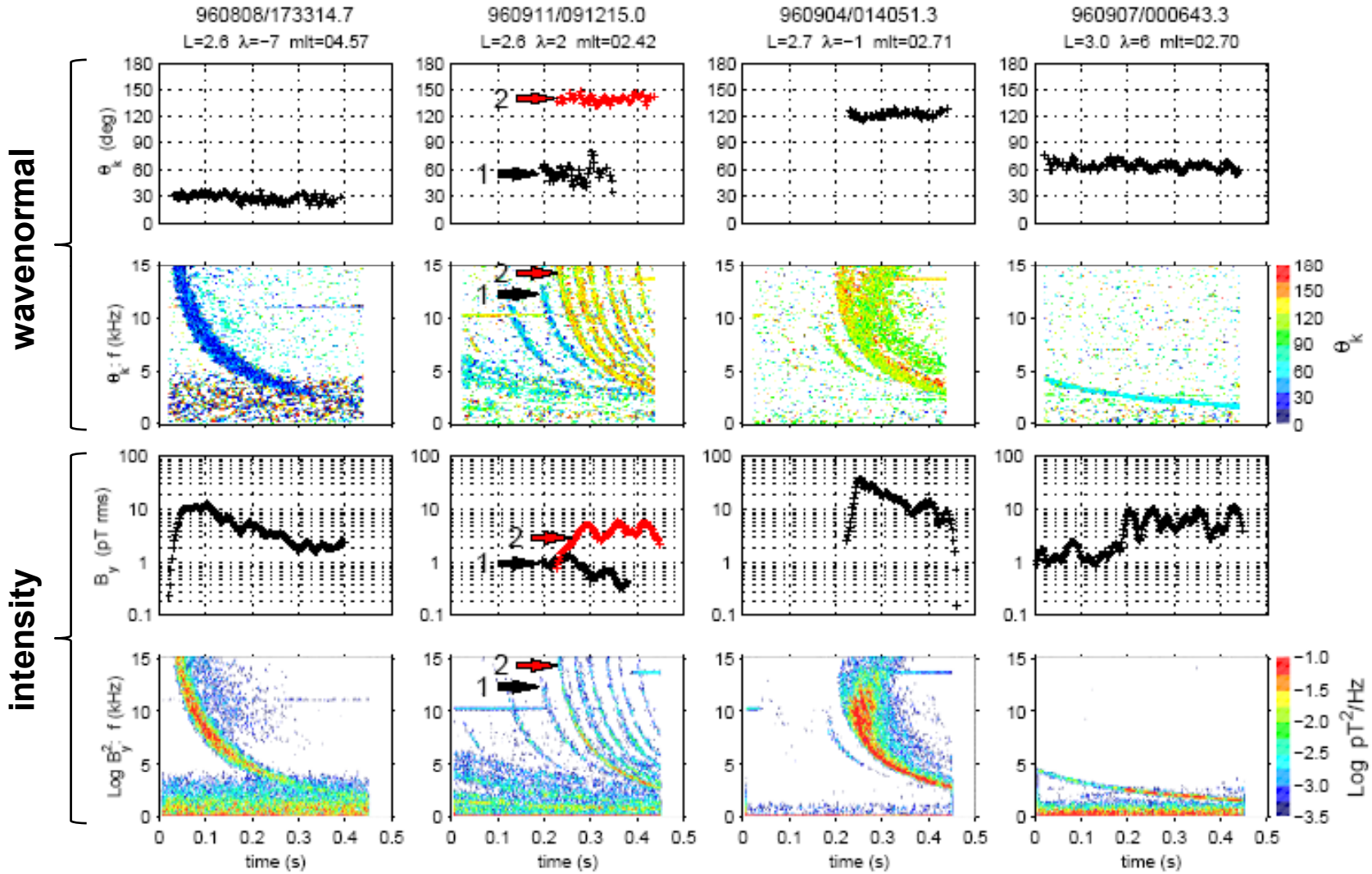
$\omega - k_{||} v_{||} = -m\,\Omega / \gamma$



Whistler range 6.5 – 12 kHz occurs frequently, yet not presently modeled
Impacts same 500 keV electrons at L=2 but with m=1 order instead of m=2

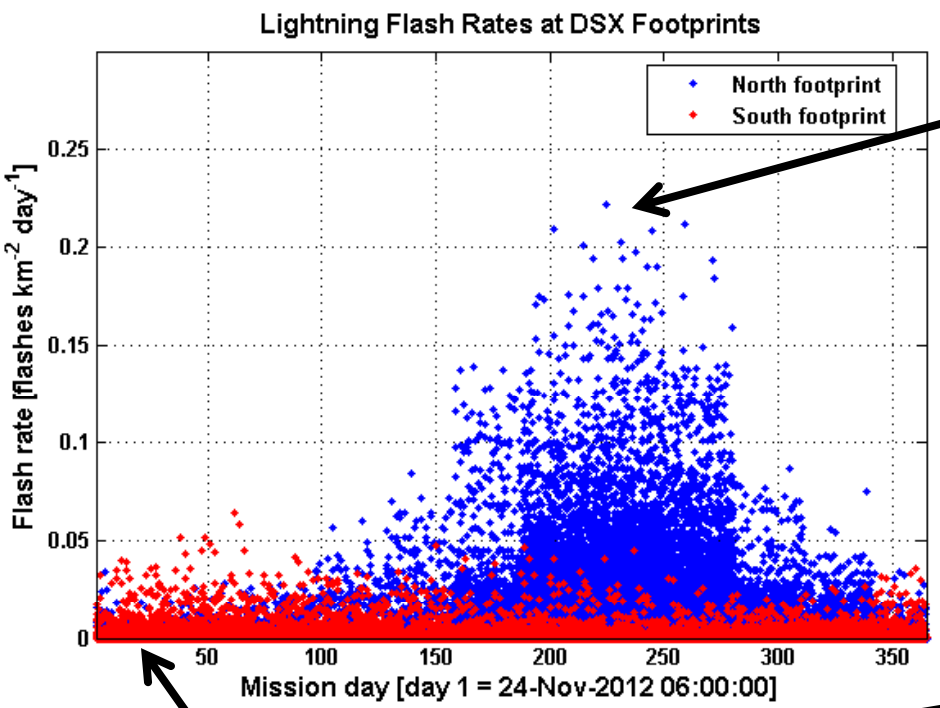
Typical Whistlers in the Slot Region

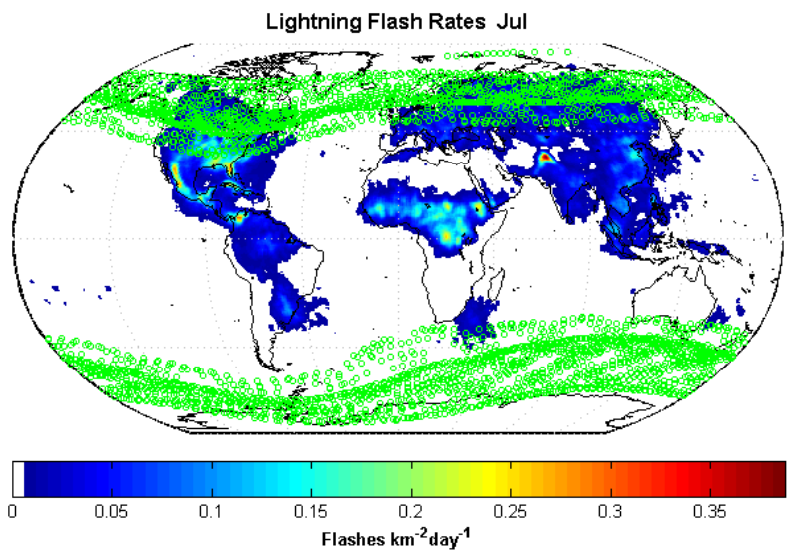
POLAR/PWI : Example waves in equatorial slot region

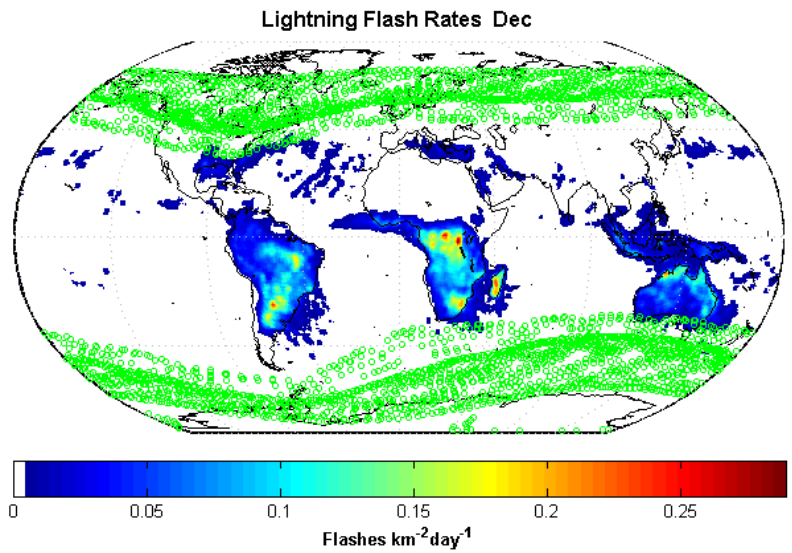


JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 106, NO. A12, PAGES 29745-29770, 2001.

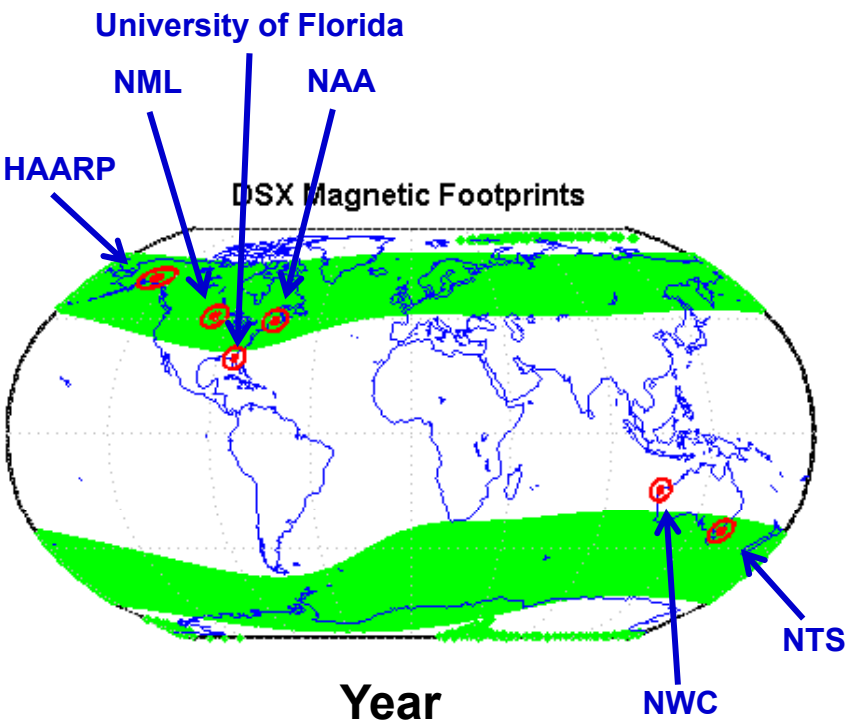
Lightning Magnetic Field Line Footprints



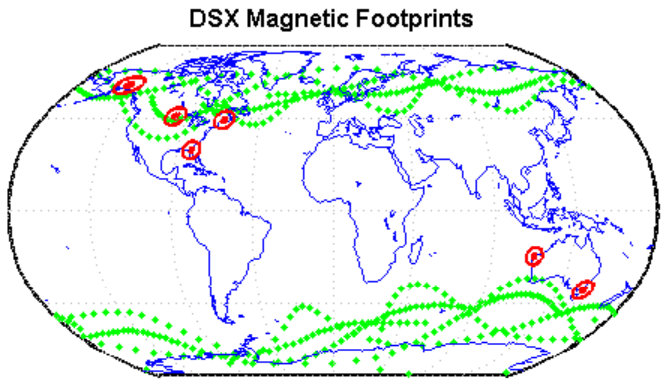




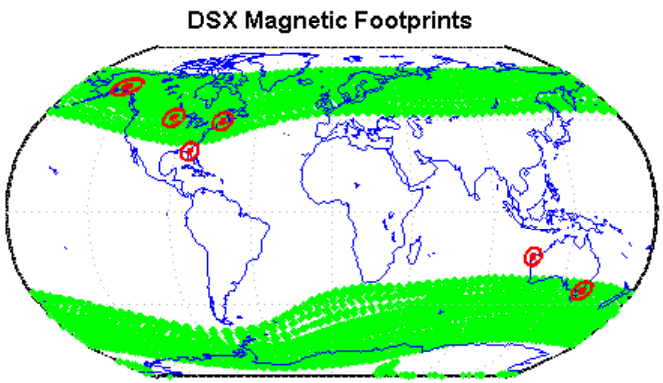
VLF Ground Station Conjunctions



Year



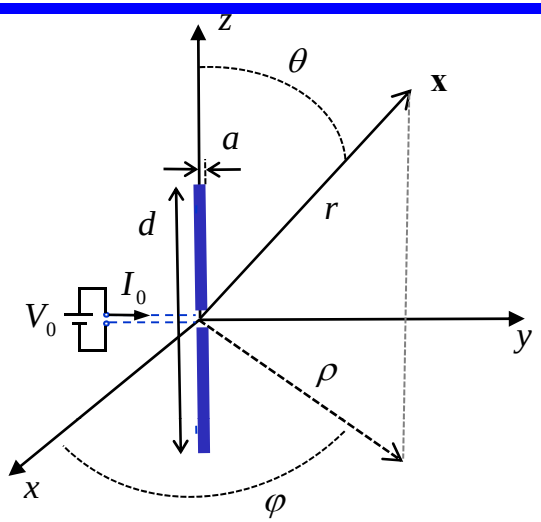
Day



Month

DSX provides ample Lightning and Ground Tx overflights for comparison

Radiation Pattern: Linear Dipole in Vacuo



Potentials: $\mathbf{E} = -\nabla\varphi - \dfrac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$

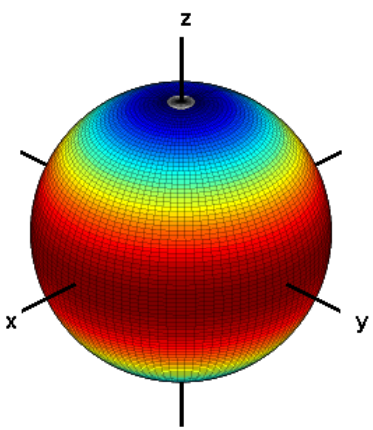
Lorentz gauge: $\nabla \cdot \mathbf{A} + \dfrac{1}{c^2}\dfrac{\partial \varphi}{\partial t} = 0$

Green's function: $\mathbf{A}(\mathbf{x},t) = \dfrac{\mu_0}{4\pi}\displaystyle\int_V d^3x' \int_{-\infty}^{t} dt' \dfrac{\mathbf{J}(\mathbf{x},t)}{|\mathbf{x}-\mathbf{x}'|}\delta\left(t' - t + \dfrac{|\mathbf{x}-\mathbf{x}'|}{c}\right)$

Boundary conditions:

$$\hat{E}_z(\rho = a) = 0, \qquad 0 < |z| < \frac{d}{2}$$

$$\hat{E}_z(\rho = a) = -\hat{V}_0\delta(z), \qquad z = 0$$



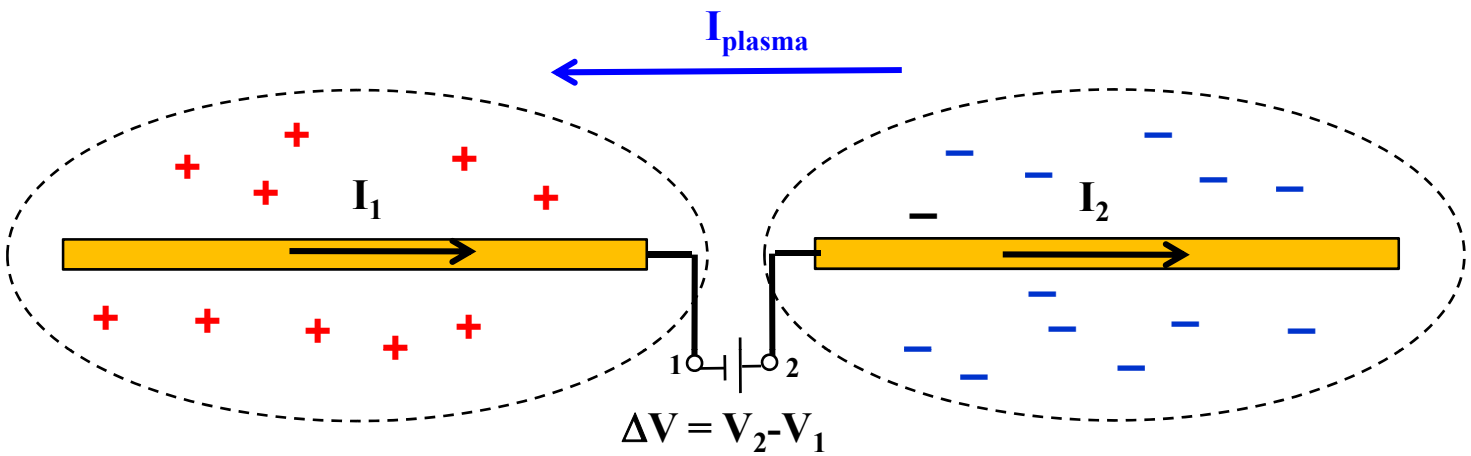
Radiation pattern

Hallén integral equation: $\left(\dfrac{\partial^2}{\partial z^2} + k_0^2\right)\displaystyle\int_{-d/2}^{d/2} dz'\, I(z',a)\dfrac{\exp\{-jk_0 r'\}}{r'} = -\dfrac{4\pi j k_0^2}{\omega}\hat{V}_0\delta(z)$

Can be solved analytically in electrically thin & short limit:

$$d << \lambda_0, \quad a << \lambda_0, \quad \lambda = \frac{2\pi}{k_0} \longrightarrow I_0 = \frac{j\pi V_0 k_0 d}{2Z_0\left[\ln\left(\dfrac{d}{2a}\right) - 1\right]}.$$

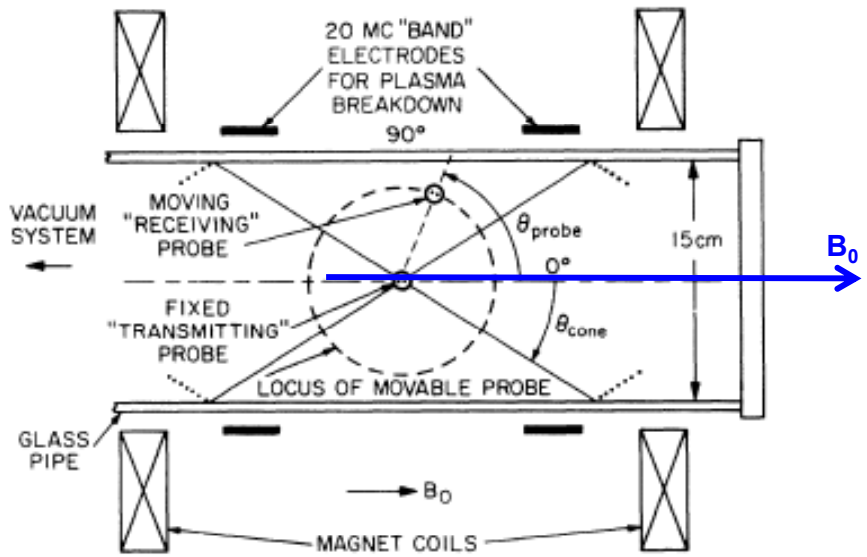
Antenna Radiation in Plasma

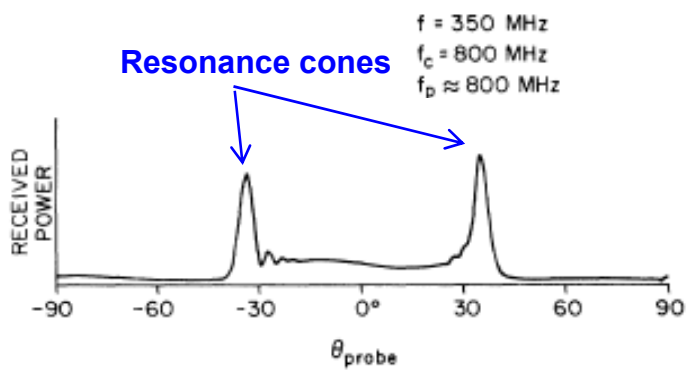


- Far-field power radiated $\sim \frac{d}{dt}\left(I_1 + I_2 + I_{plasma}\right)$
- Sheaths can change antenna reactance
- Antenna fields heat local plasma
- Anisotropic medium can dictate complicated far-field radiation pattern

Radiation in Plasma: Resonance Cones

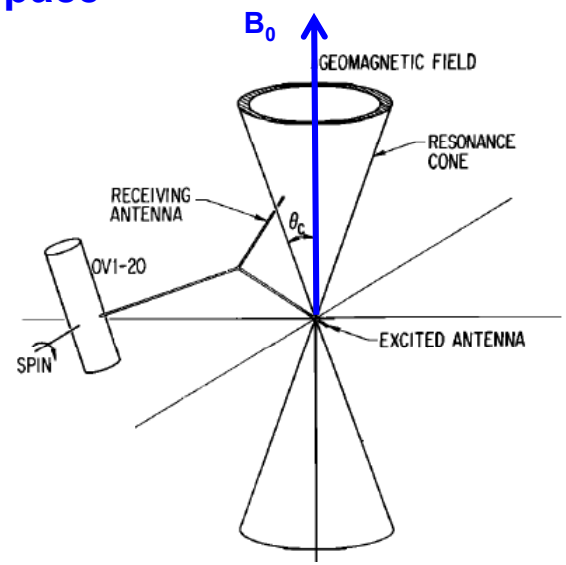
In the laboratory

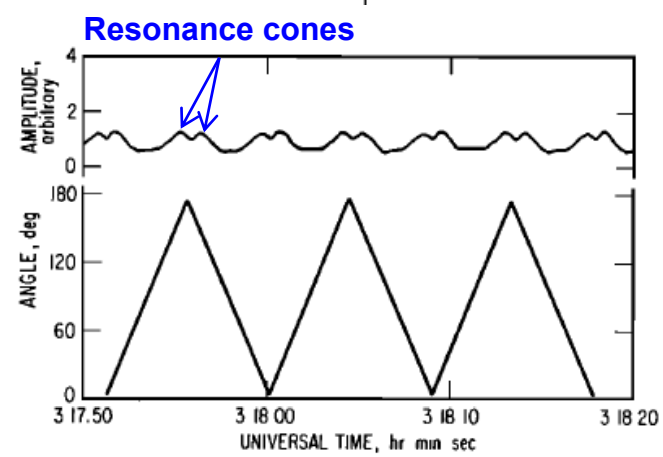




Fisher and Gould, Resonance Cones in the Field Pattern of a Short Antenna in an Anisotropic Plasma, *Phys. Rev. Lett.*, 22, 1092-1095, 1969.

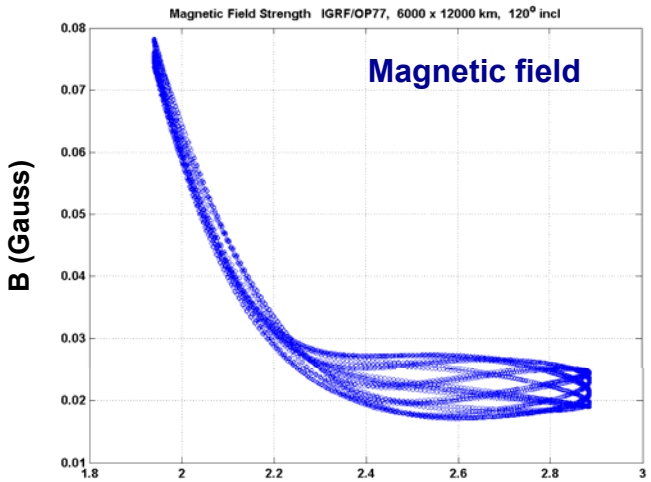
In space

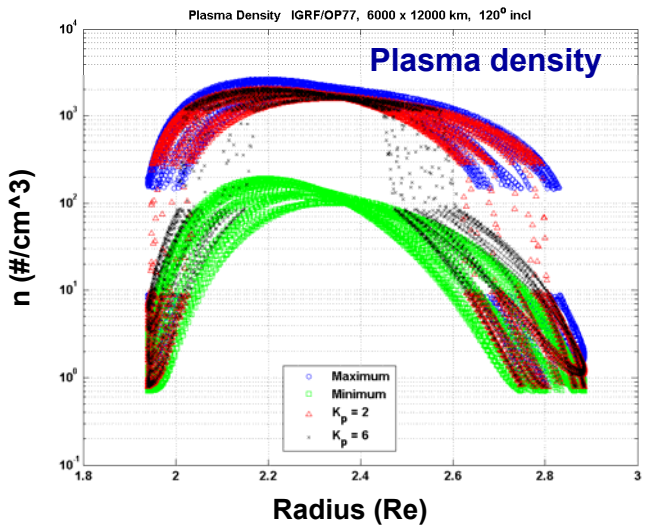


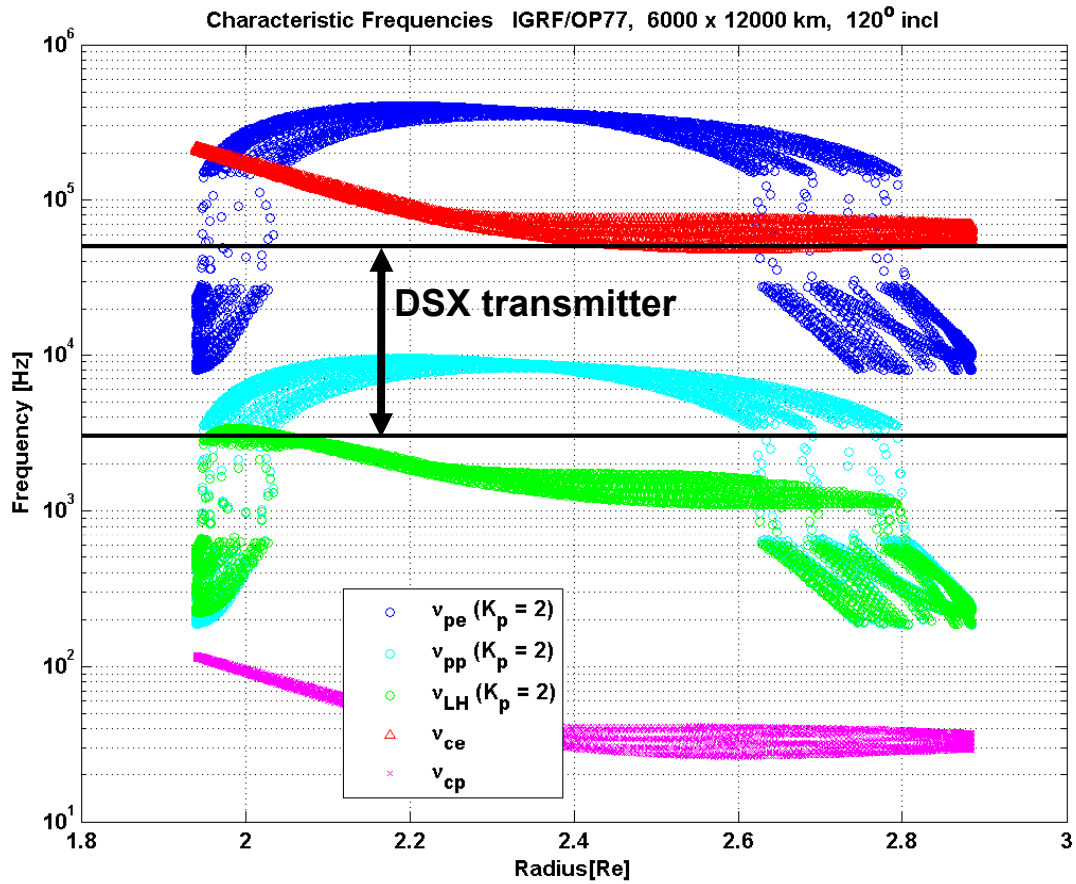


Koons, et al., Oblique resonances excited in the near field of a satellite-borne electric dipole antenna, *Radio Sci.*, 9, 541-545, 1974.

Plasma Environment







Characteristic frequencies

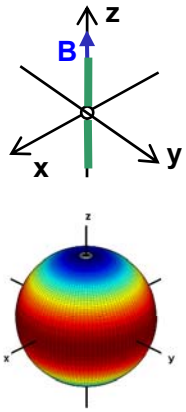
Stix S, D, P, R, L → $R_{rad}(k_x, k_y, k_z)$

DSX Radiation Patterns

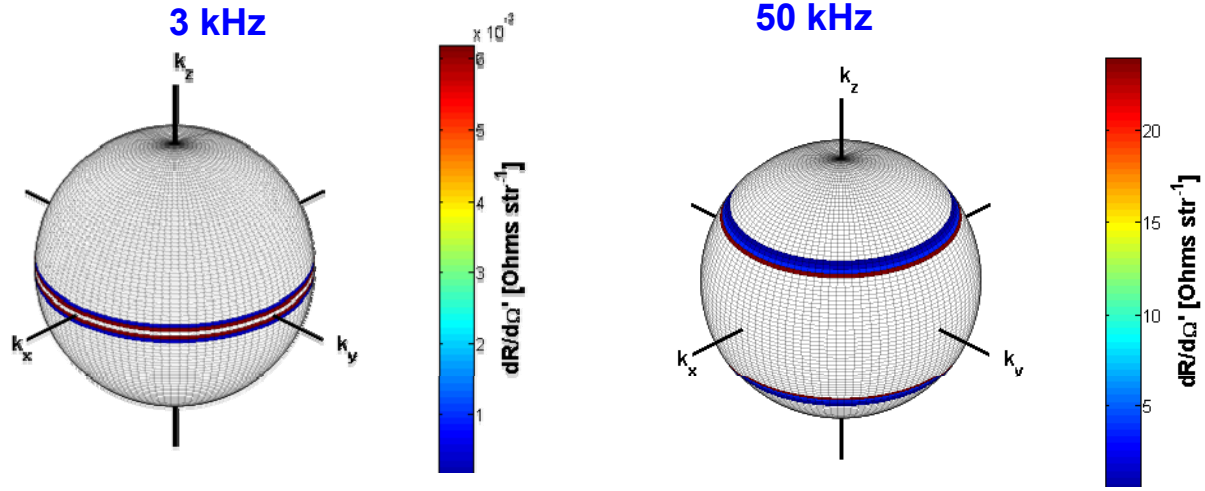
Inside the plasmasphere

3 kHz **50 kHz**

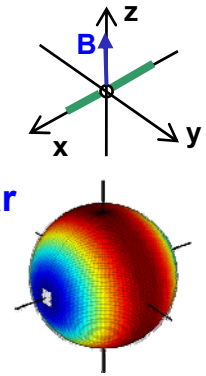
Parallel



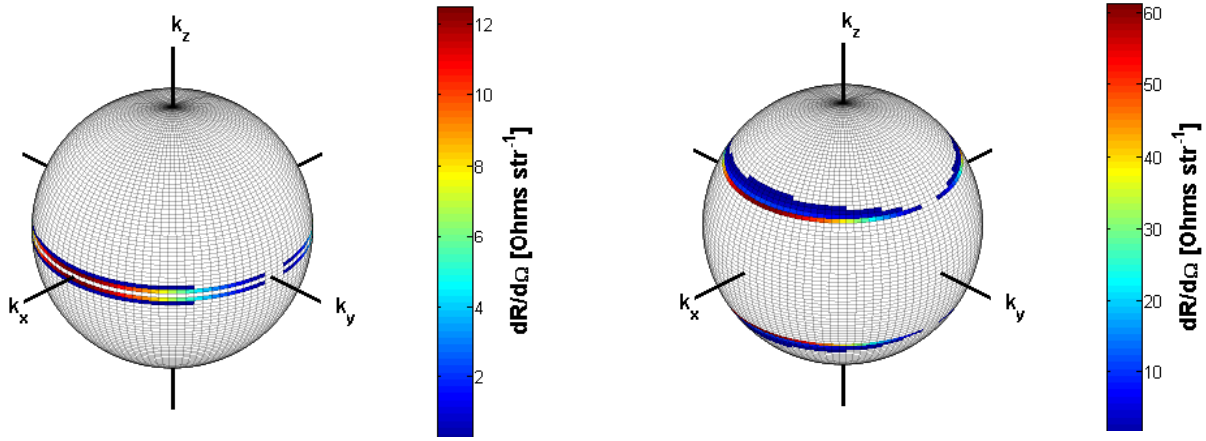
vacuum



Perpendicular



vacuum

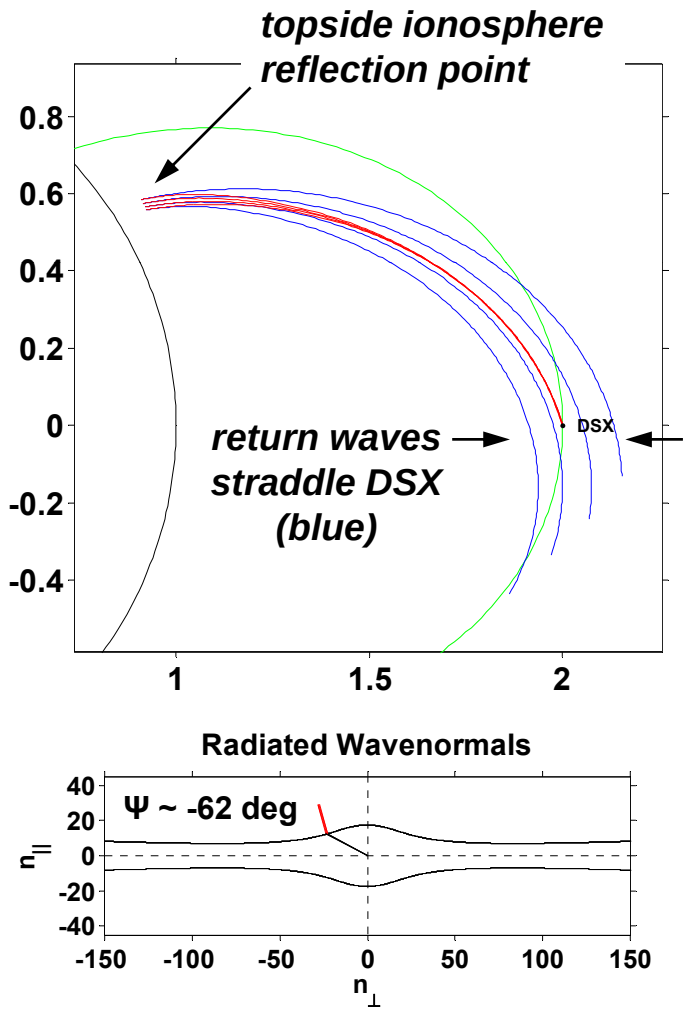


θ_{cutoff} = 89.4° – 68.3°, ν = 3 – 50 kHz

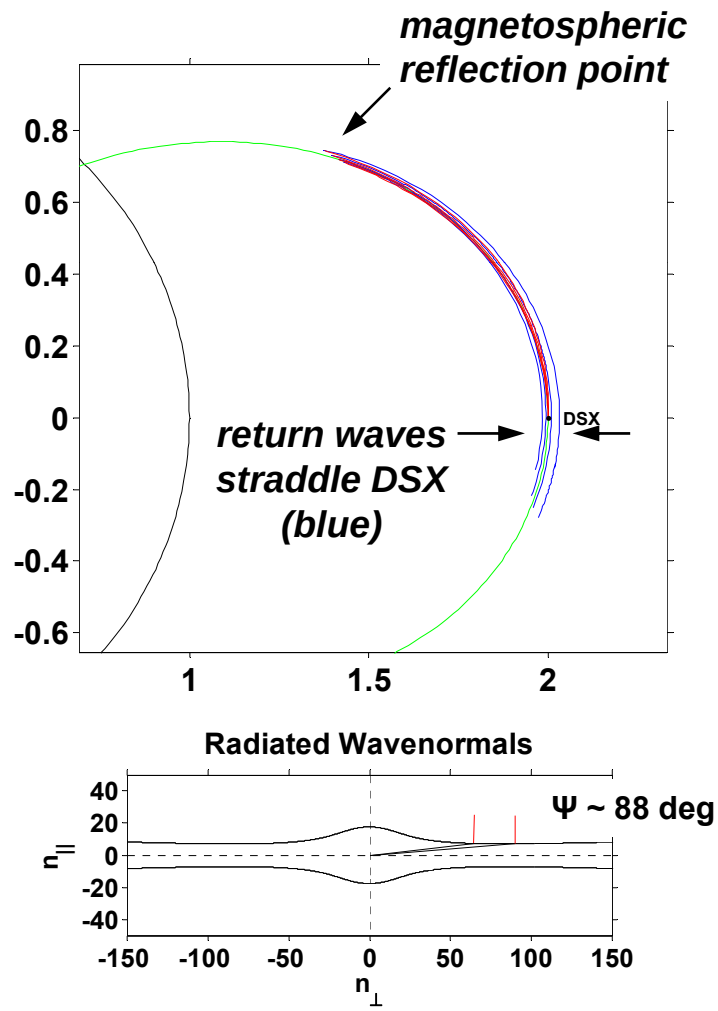
Magnetoplasma Strongly Controls Far-Field Whistler-Mode Radiation Pattern

DSX Pattern Self-Measurement

Atmospheric Reflection



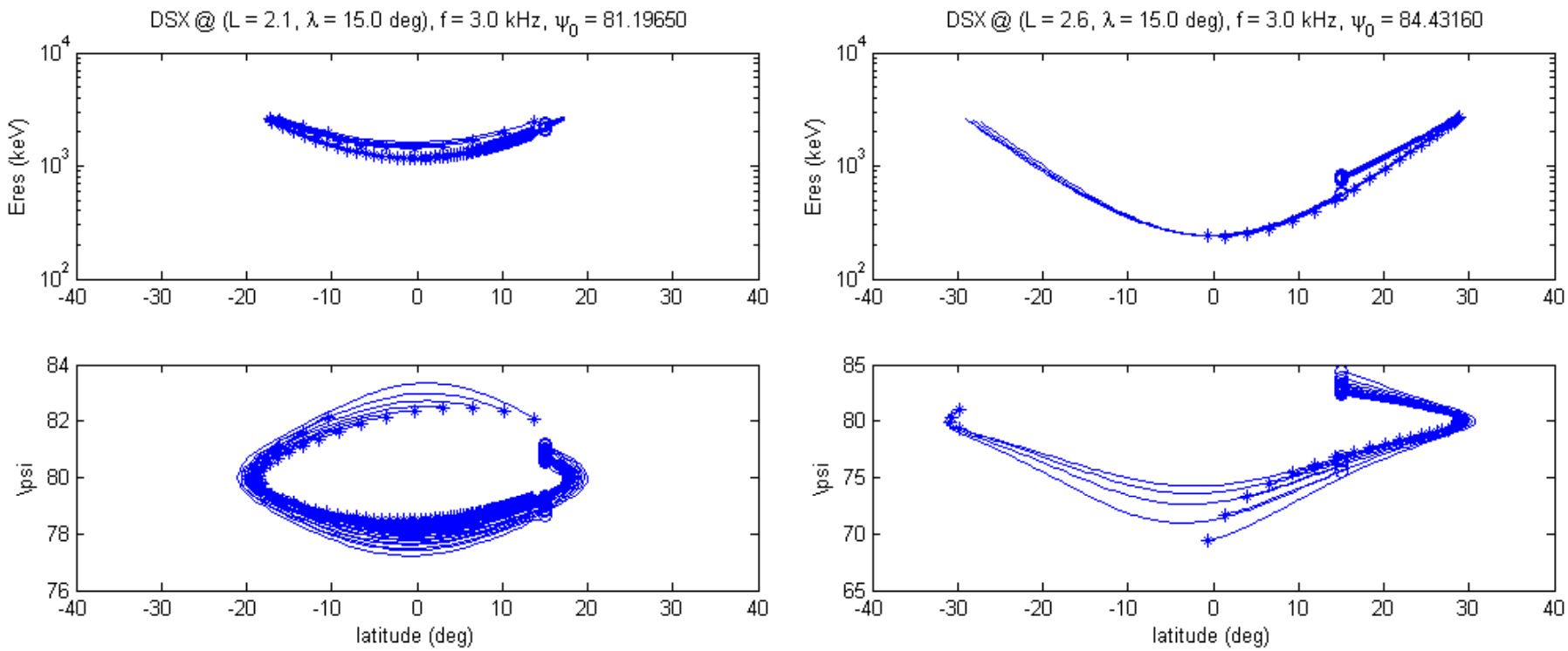
Boomerang Return



What Resonant Energies?

Evaluate $\omega - k_{||} v_{||} = -m\,\Omega / \gamma$ along ray paths to solve for $v_{||}$

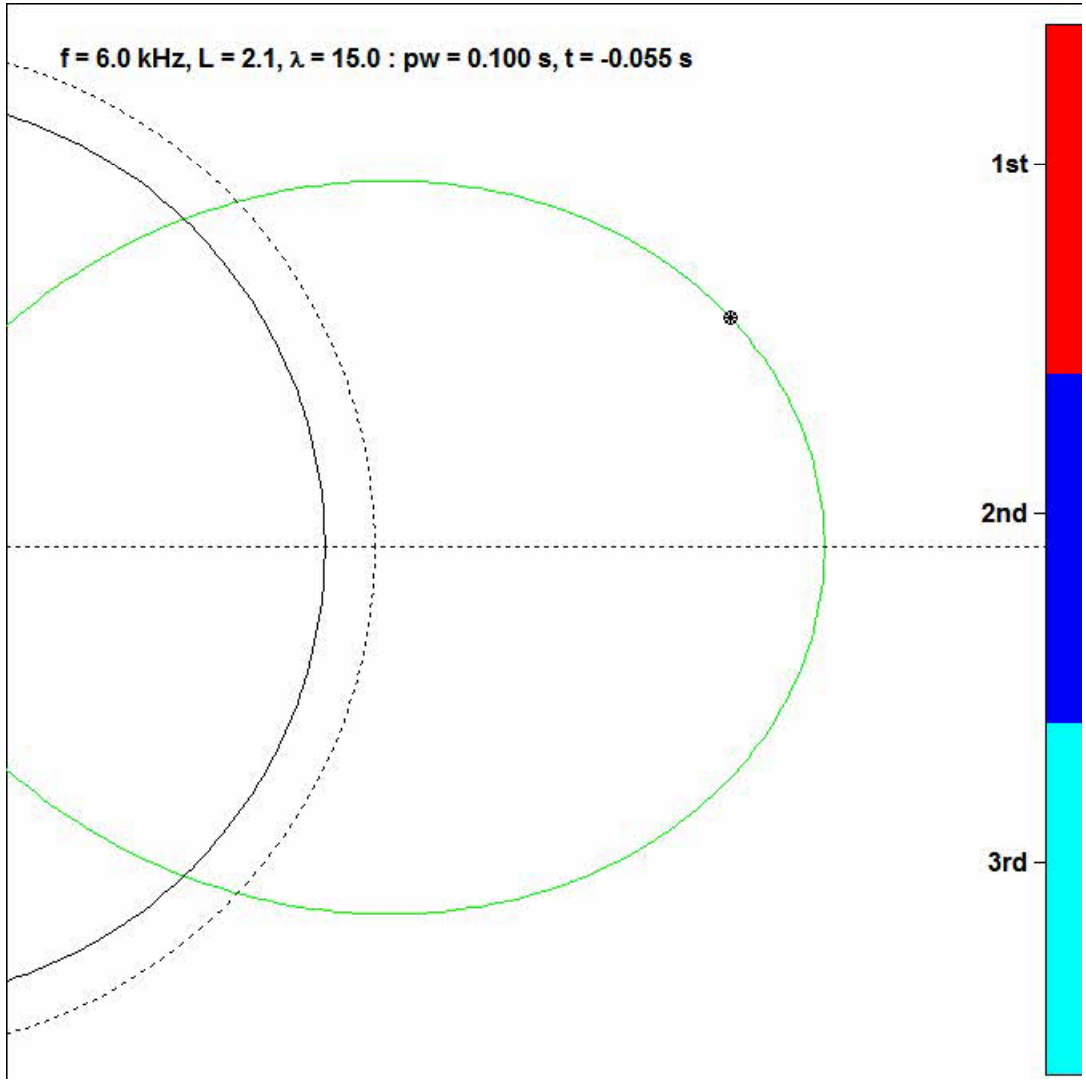
L = 2.1 ***Preliminary!*** **L = 2.6**



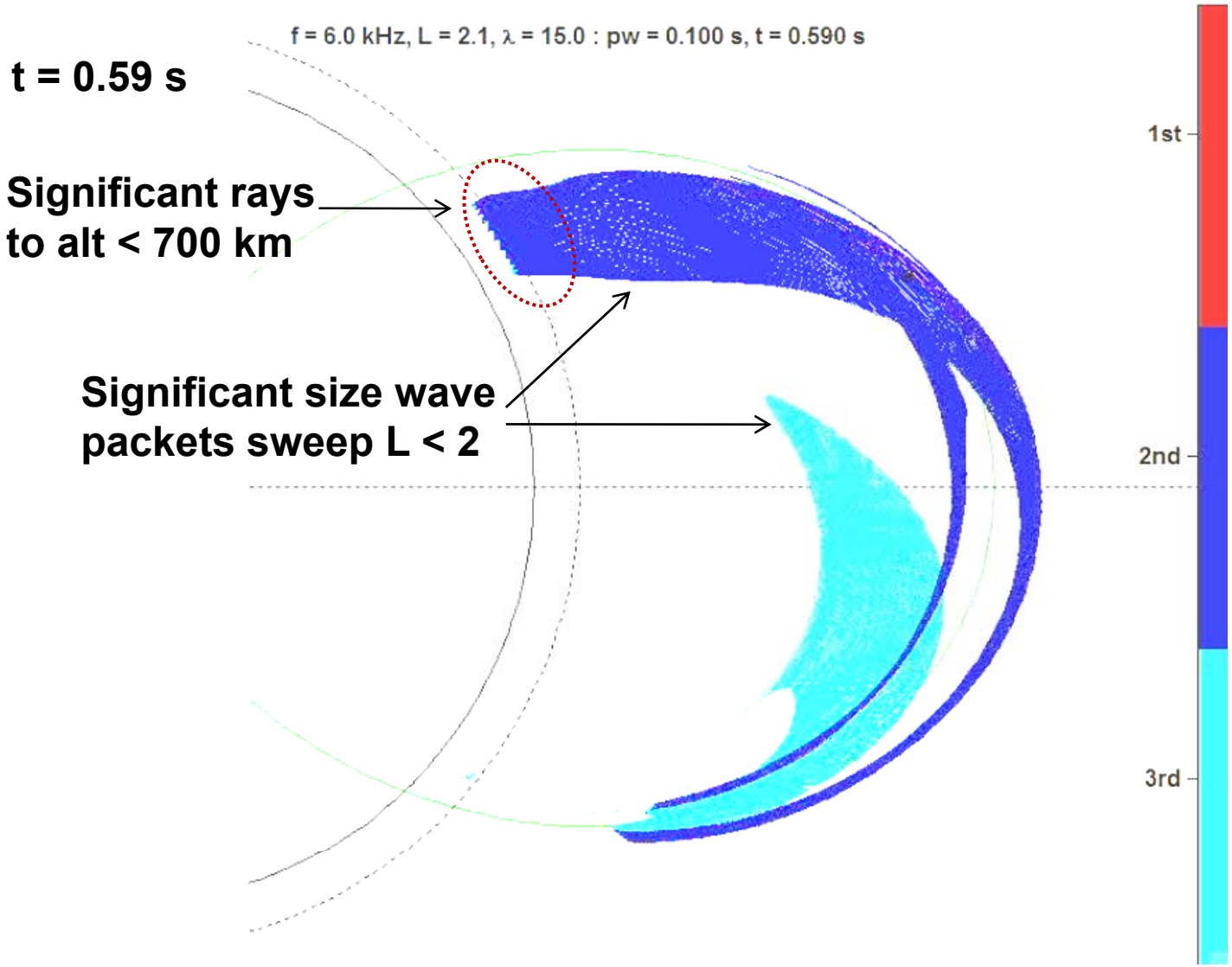
DSX Far-Field Propagation(1)

- But, whistler ray paths are NOT LINE OF SIGHT!
- They're not generally field-aligned, either!

- This animation shows a 0.1 sec pulse of f = 6 kHz rays for duration 2.0 sec
- Rays launched w/in 3 deg of local resonance cone, at L = 2.1, mlat = 15 deg
- The 1st pass rays = red, 2nd = blue, 3rd = cyan

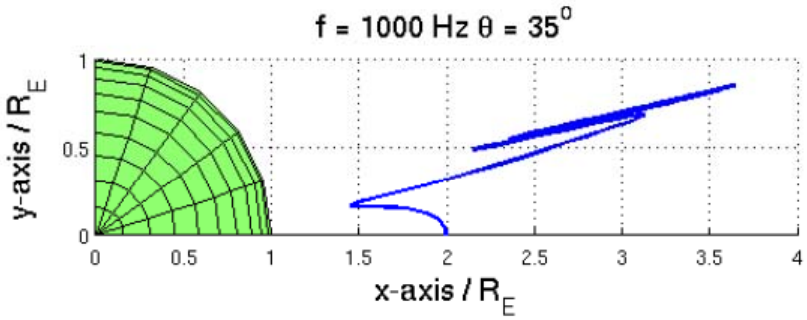


DSX Far-Field Propagation(2)

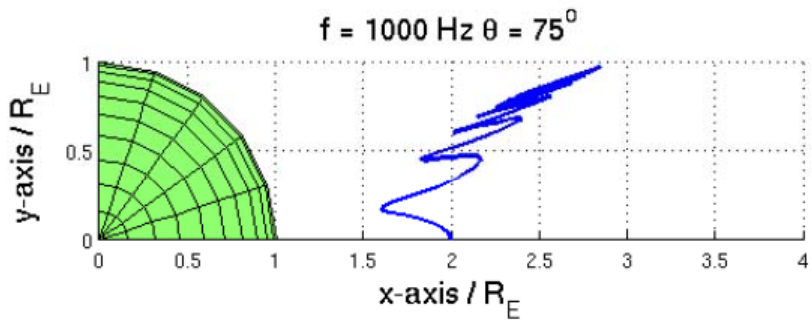


Extending to 3D Propagation…

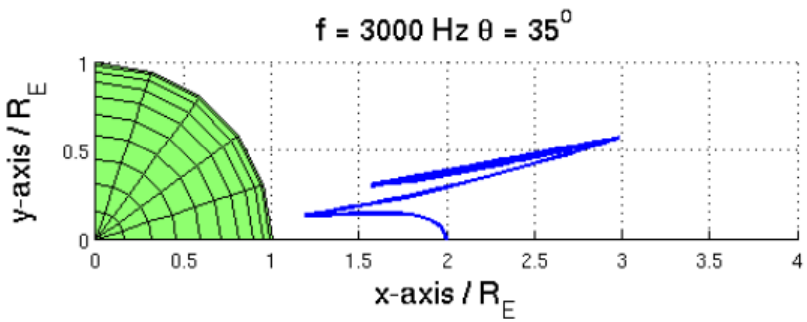
Wave energy settles (and damps) in longitudinal "wings"
Morphology is frequency and wavenormal dependent



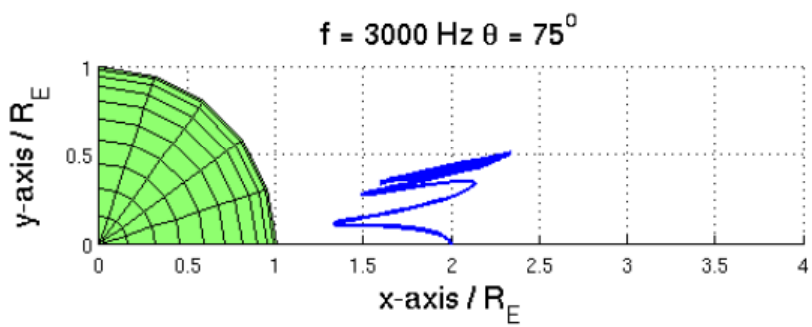
(a) f = 1000 Hz $\theta = 35^o$



(b) f = 1000 Hz $\theta = 75^o$



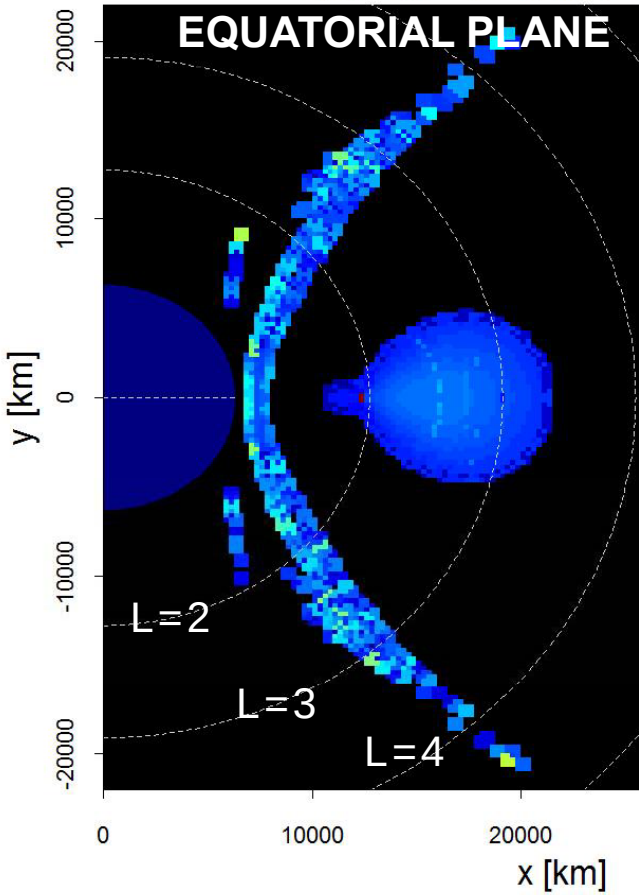
(c) f = 3000 Hz $\theta = 35^o$

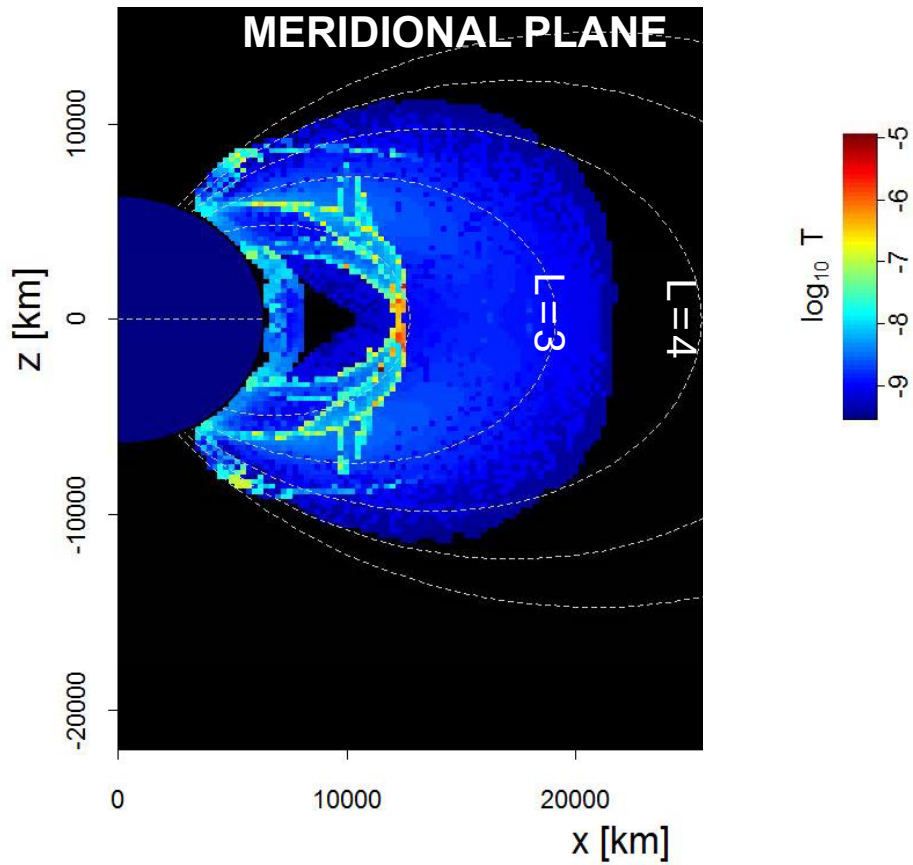


(d) f = 3000 Hz $\theta = 75^o$

DSX Global Power Distribution – 3 kHz

Satellite at 6000 km altitude, 0° magnetic lat, vacuum antenna limit; 3 kHz

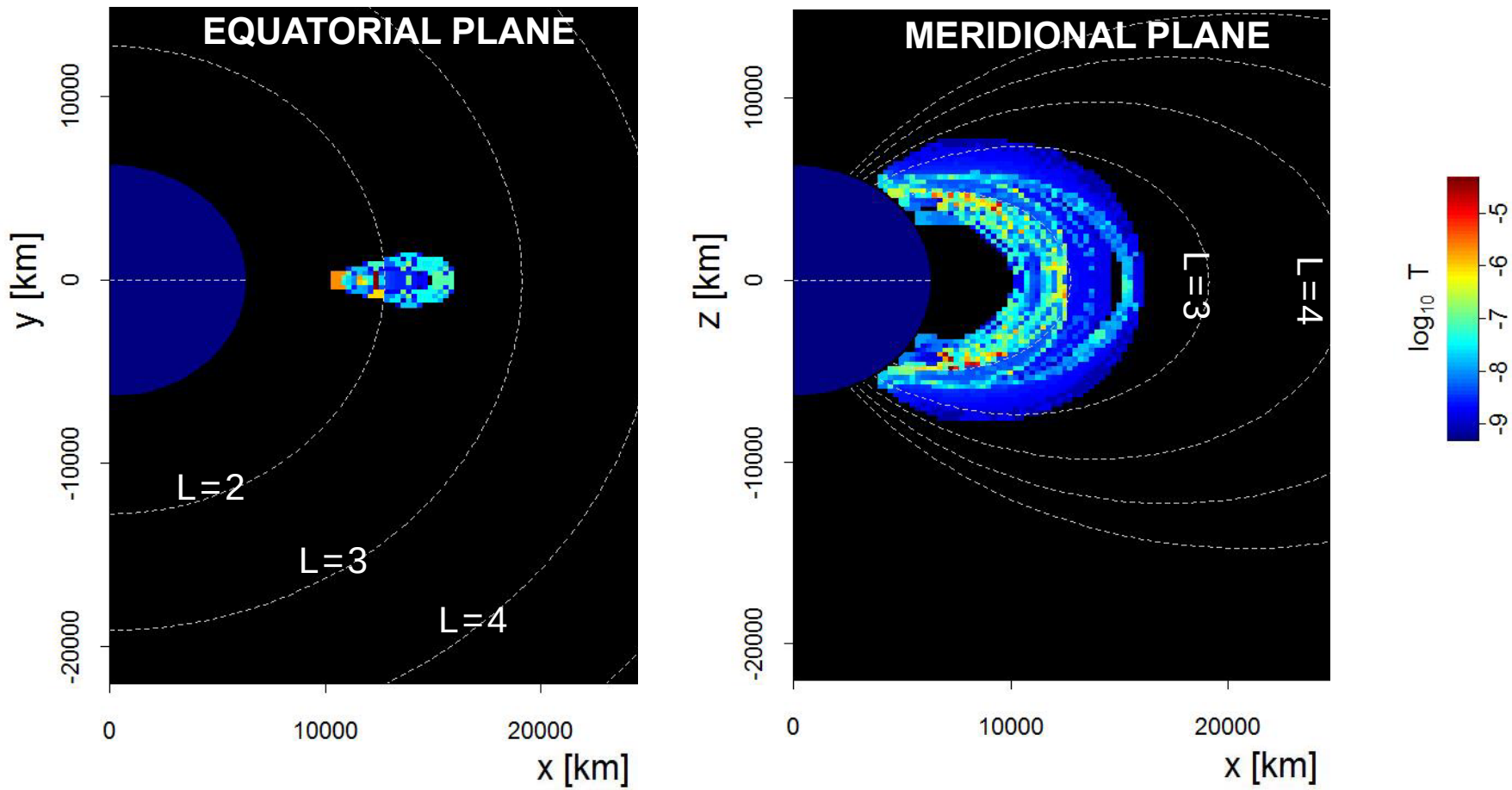




Critical Unknown:
Importance of scattering and mode-conversion on power and k-spectrum

DSX Global Power Distribution – 10 kHz

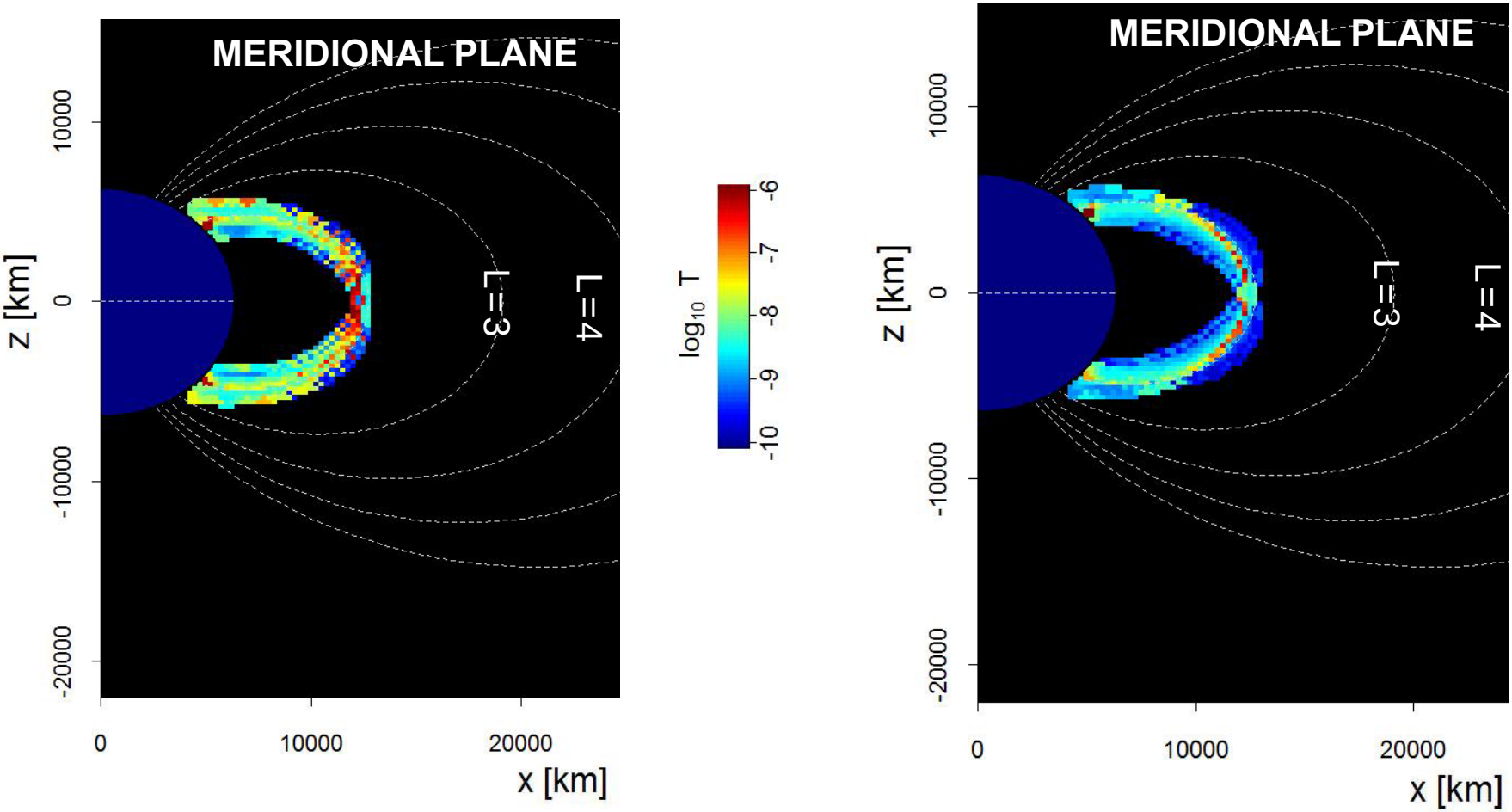
Satellite at 6000 km altitude, 0° magnetic lat, vacuum antenna limit; 10 kHz



Magnetospheric reflection destines rays for lower altitudes at 10 kHz as compared to 3 kHz

DSX Global Power Distribution – 12 kHz

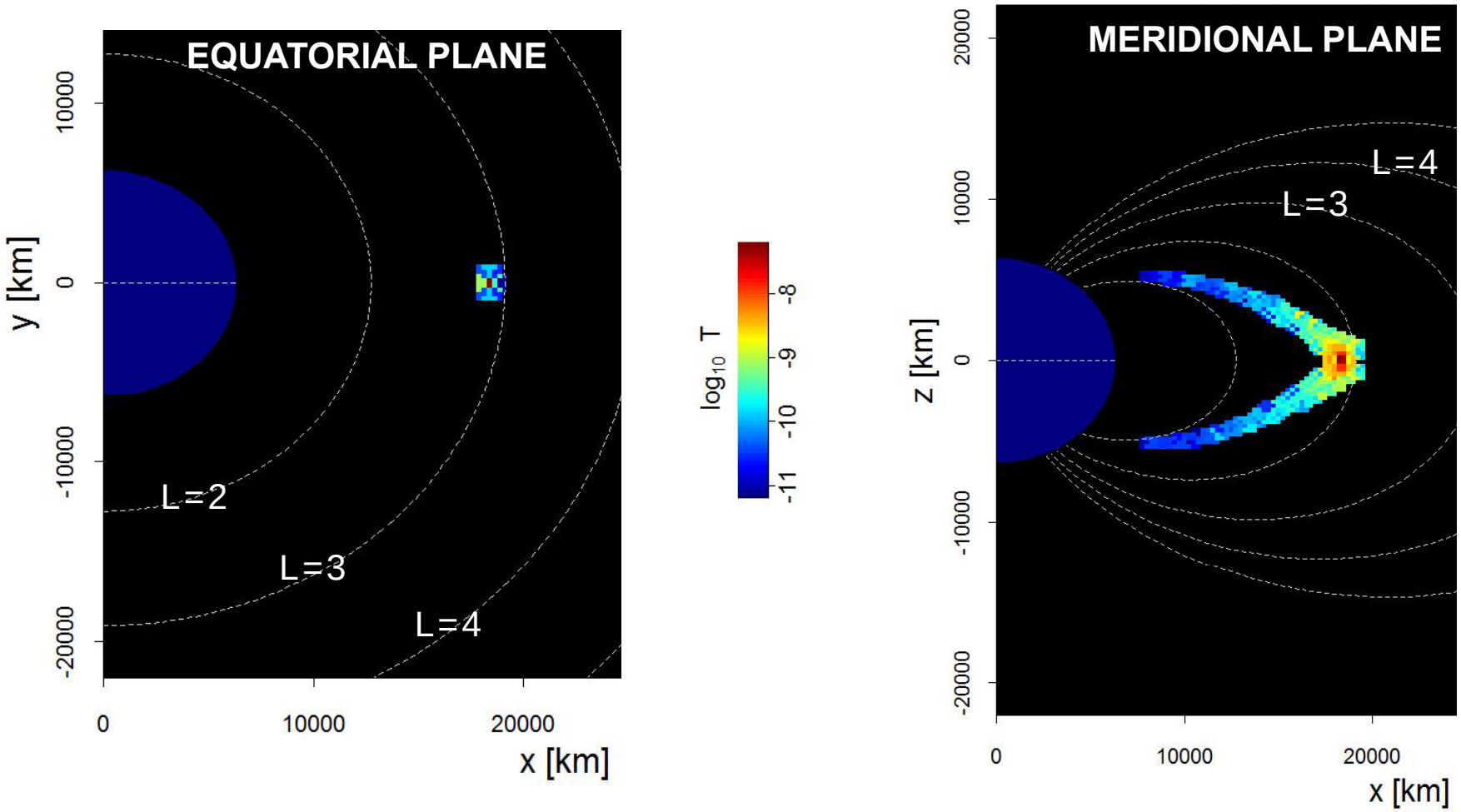
Satellite at 6000 km altitude, 0° magnetic lat, vacuum antenna limit; 12 kHz



Magnetospheric reflection is not a factor above about 12 kHz. Transmitter energy is concentrated around the field line of the transmitter.

DSX Global Power Distribution – 30 kHz

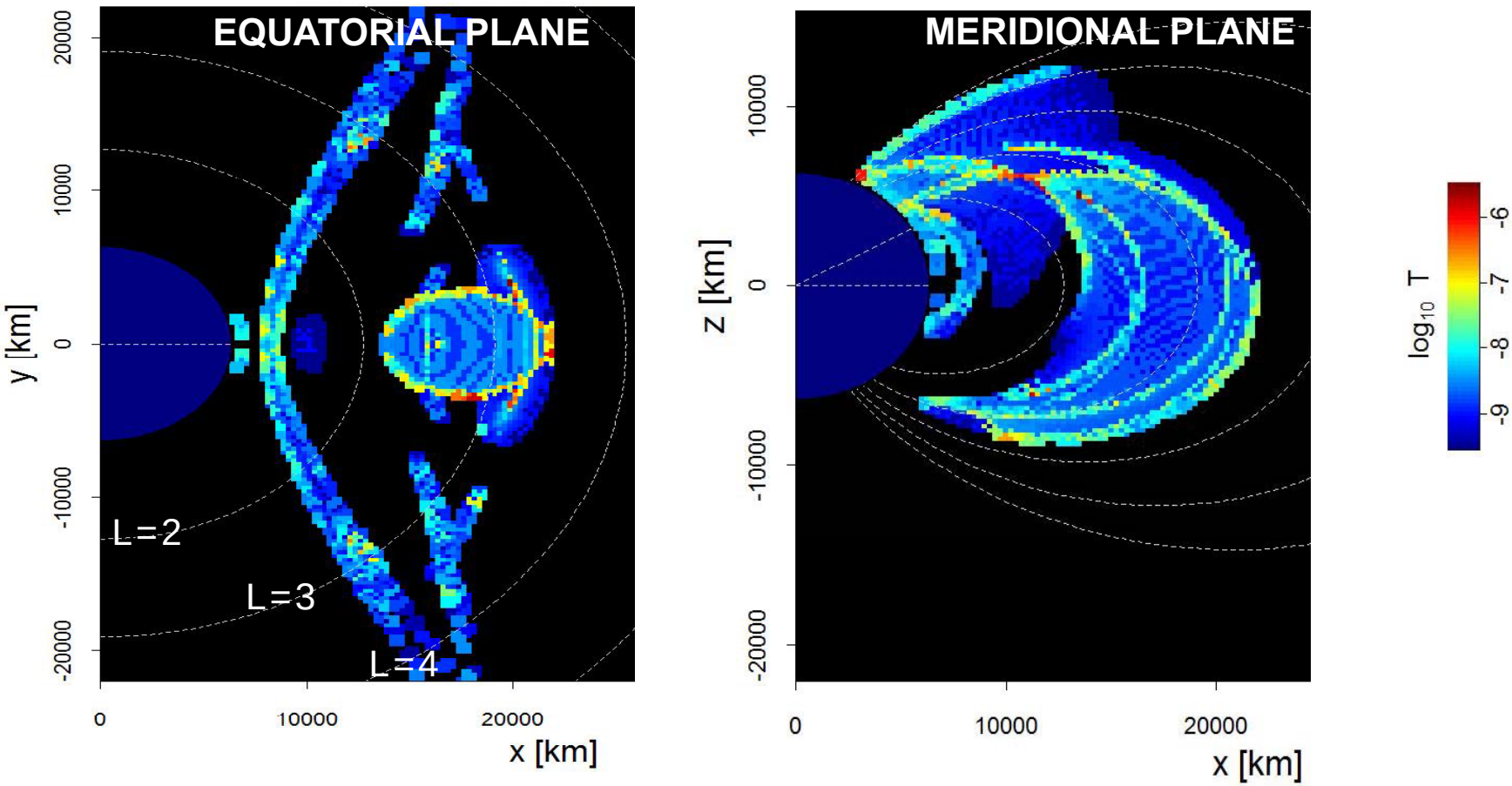
Satellite at 12000 km altitude, 0° magnetic lat, vacuum antenna limit; 30 kHz



At high altitude, 30 kHz does not propagate well, losing intensity extremely quickly.

DSX Global Power Distribution – Off Equator

Satellite at 6000 km altitude, 30° magnetic lat, vacuum antenna limit; 10 kHz

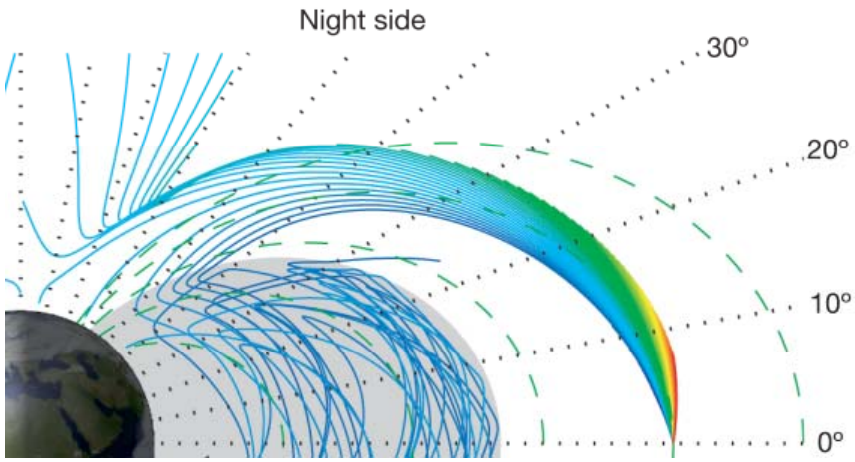


The off-equatorial transmitters lead to very complex field distributions.

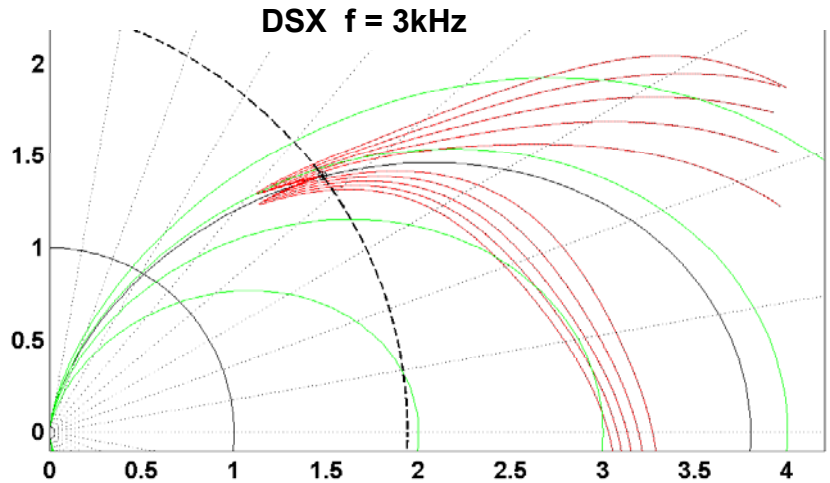
Joint Experiment Opportunities – Space

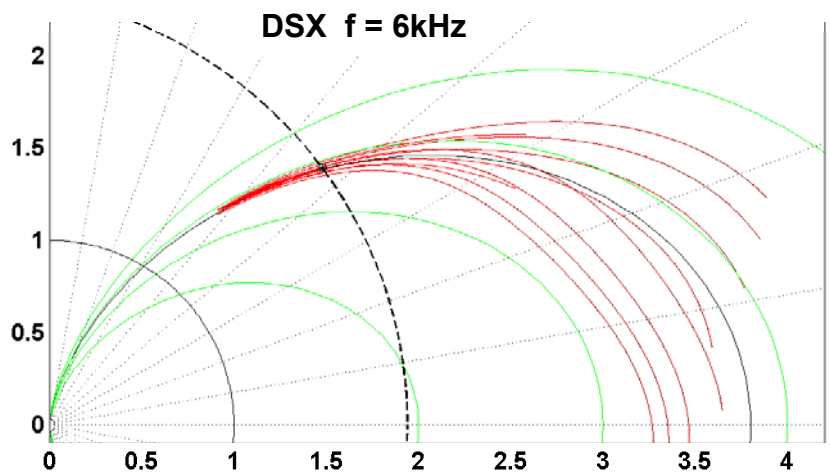
- **Cassiope/Enhanced Polar Outflow Probe (E-PoP), CSA, CRC (James), NRL (Siefring, Bernhardt)**
  - 300 x 1500 km, polar inclination, launch 2011 (?)
  - Radio Receiver Instrument (RRI), ELF-VLF 10 Hz -30 kHz, two-axis E-field
  - Fast Auroral Imager (FFI), ~ 1 MeV electrons
- **Radiation Belt Storm Probes (RBSP), NASA**
  - 2 satellites in GTO, < 18 deg incl, launch no earlier than fall 2011
  - Electric and Magnetic Field Instrument Suite and Integrated Science Suite (EMFISIS, Univ. of Iowa, Kletzing), 3 axis B-field, 2 axis E-field 10 Hz – 12 kHz (1 channel E-field 10 kHz – 400 kHz)
  - Magnetic Electron-Ion Spectrometer (MagEIS, BU & Aerospace, Spence & Blake), 40 keV – 10 MeV electrons
  - Relativistic Electron-Proton Telescope (REPT, BU & Univ. of Colorado, Spence & Baker), 2 MeV – 10 MeV electrons
  - RBSP Ion Composition Explorer (RBSPICE, NJIT, Lanzerotti), 25 keV – 500 keV electrons
- **TARANIS, CNES, Stanford Co-PI (Inan), follow on to DEMETER**
  - 700 km, polar, launch 2011(?)
  - IMM-MF, B-field 3 component, ~2 Hz – 20 kHz, 1 component 10 kHz – 1MHz
  - IDEE, electron detectors, 70 keV – 4 MeV
- **VPM, AFRL (Starks), Stanford (Linscott)**
  - Cubesat in ~700 km, high-inclination LEO
  - VLF receiver & loss-cone electron detector
  - Approved by AFRL in Feb 2011 for development & launch in DSX timeframe
- **ORBITALS, CSA, Univ. of Calgary (Mann), Univ. of Colorado (Baker)**
  - GTO, launch (?)
  - SCM, B-field up to 20 kHz
  - EPS, electrons 25 keV – 12 MeV

Active Injection : Test Chorus → Hiss



[Bortnik et al., Nature 452, 6 March 2008]

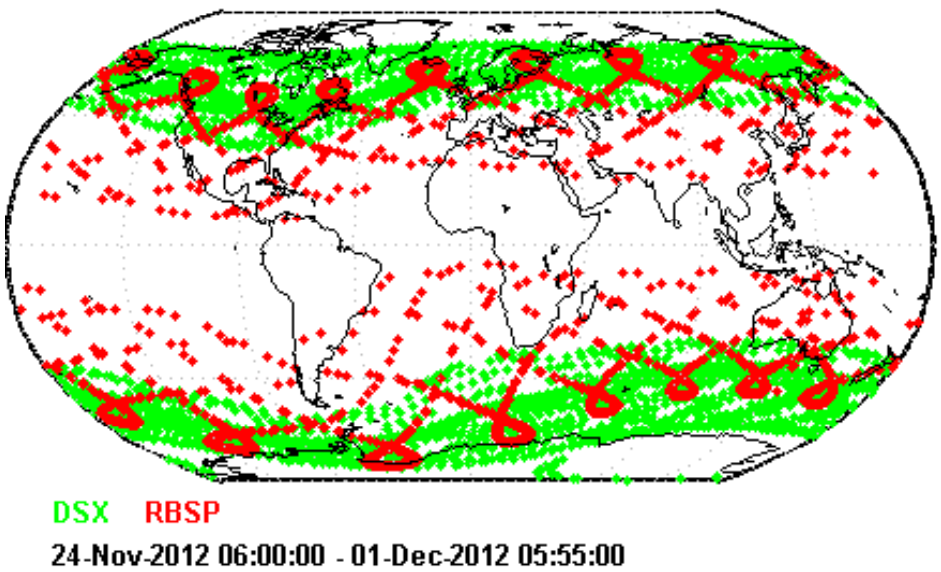




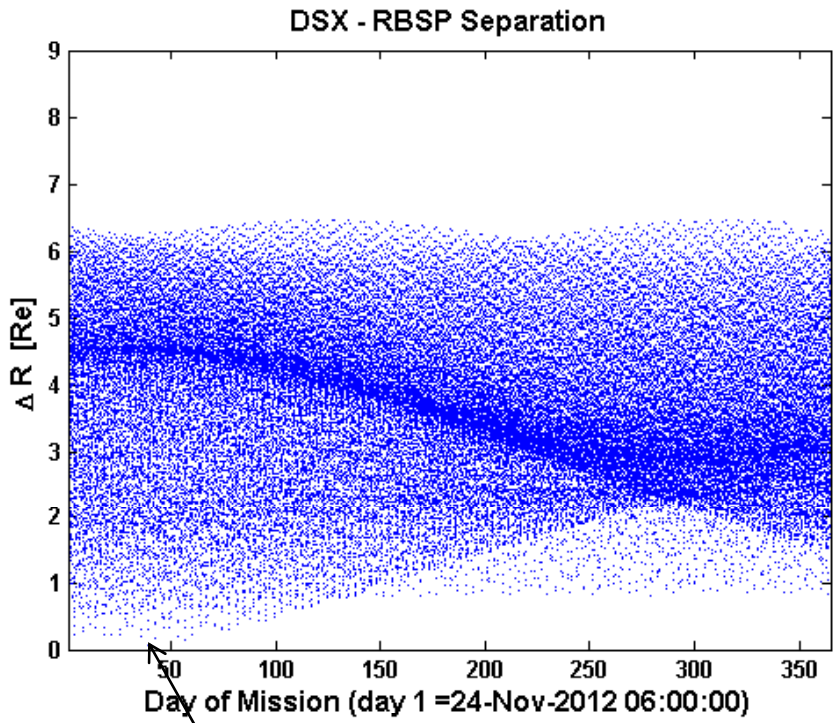
DSX Can Inject "Just Above" the Chorus → Hiss Entry Points

DSX – RBSP Conjuctions

Magnetic footprints

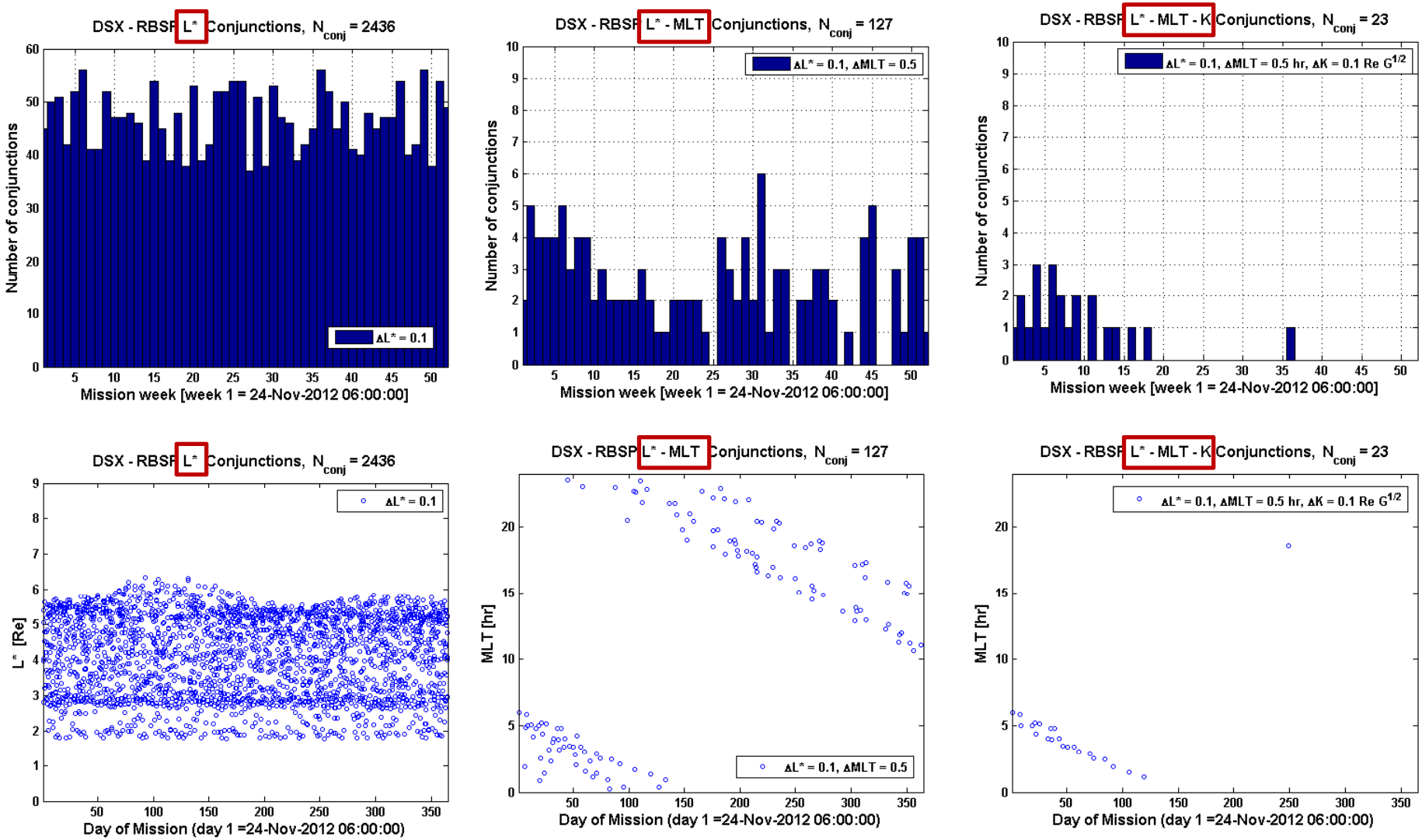


1 week (typical)



Closest approach ~ 422 km

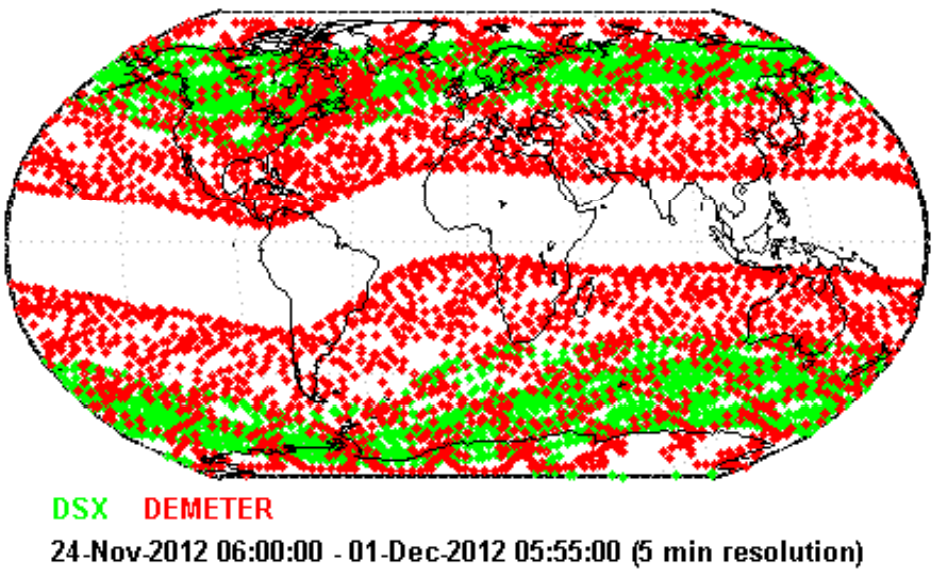
DSX – RBSP Conjunction Occurence



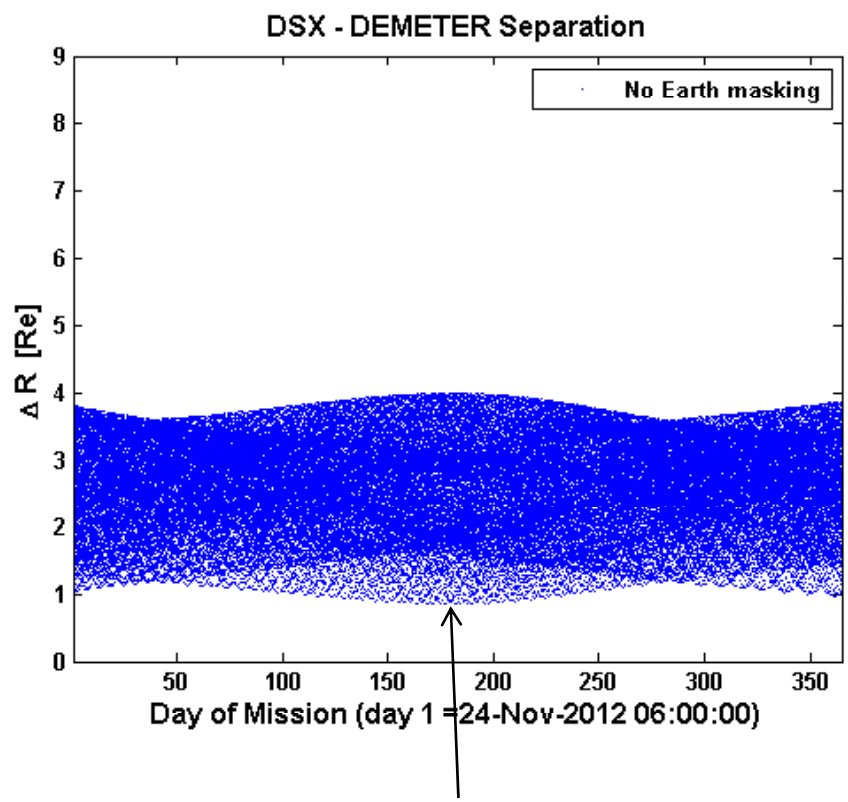
L shell **Magnetic field line** **Magnetic point**

DSX – LEO (Demeter-like) Conjunctions

Magnetic footprints

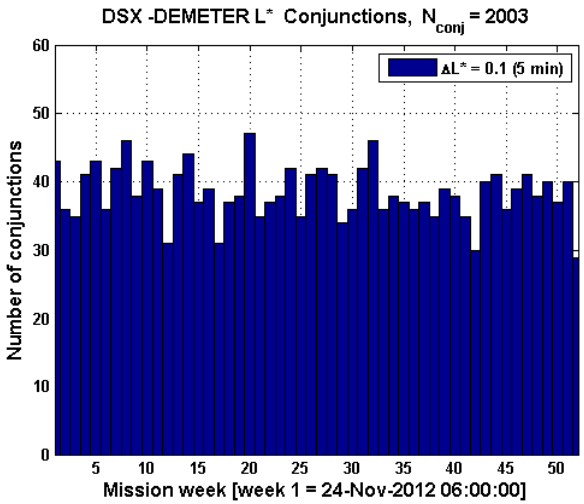


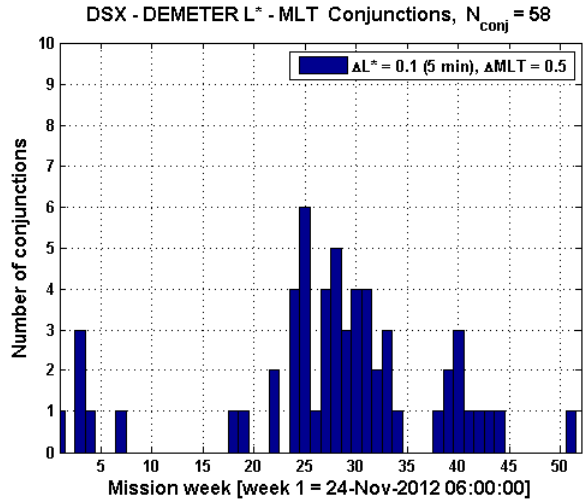
1 week (typical)

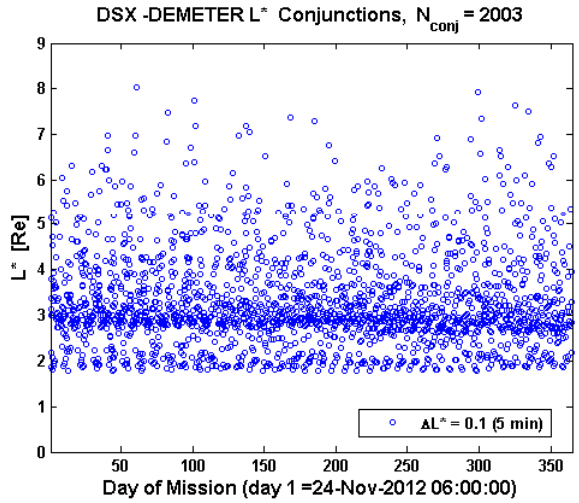


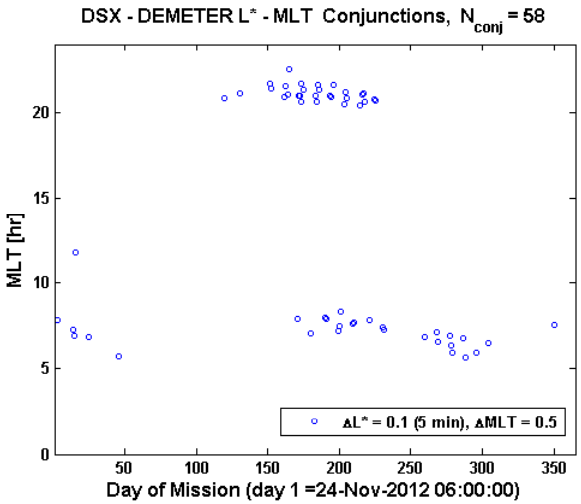
Closest approach ~ 5393 km

DSX – LEO Conjunction Occurence









None

L shell

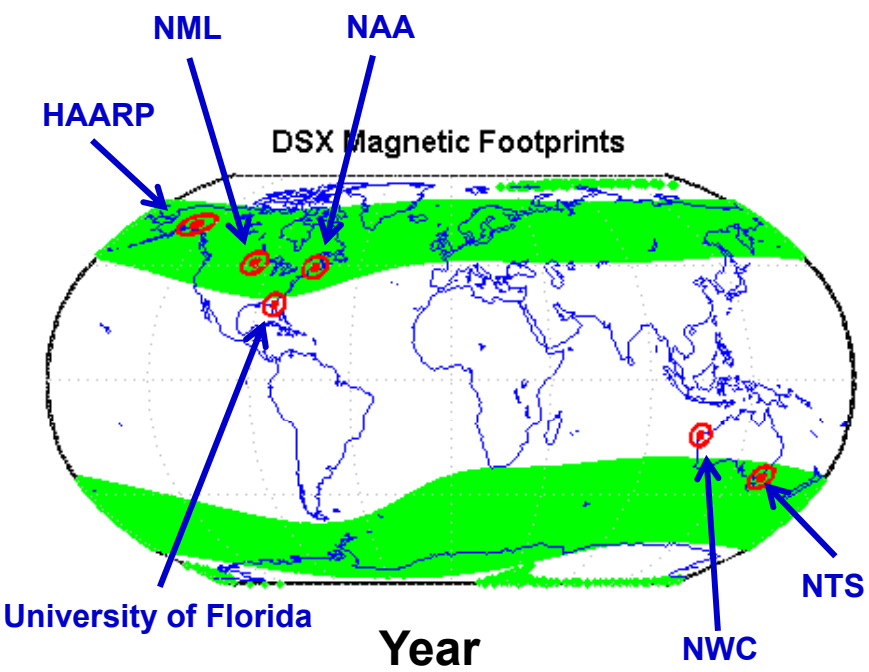
Magnetic field line

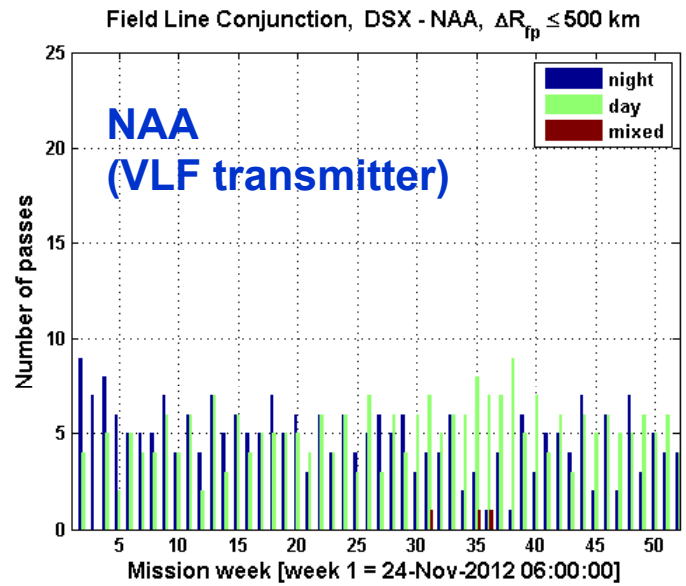
Magnetic point

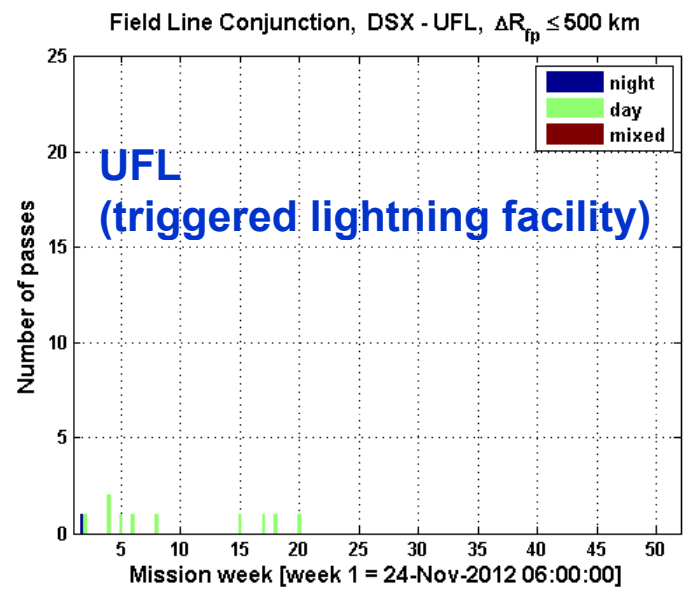
Joint Experiment Opportunities – Ground

- **High-Frequency Active Auroral Research Program** (HAARP, AFRL)
  - Electrojet-modulated VLF antenna at L ~ 4.8 with extensive frequency & mode control
- **DoD VLF transmitters**, TIPER program (AFRL & Stanford)
  - Keyed transmissions from NWC at Churchill, Australia, L ~ 1.3, 19.8 kHz, 1 MW
  - Mobile VLF transmitter broadcasts, ~ 18 kHz
- **Balloon Array for Radiation-belt Relativistic Electron Losses** (BARREL)
  - Measurement of precipitating MeV electrons at high latitudes with 5-8 balloon flotilla aloft for ~ one month

DSX – Ground Station Conjunctions





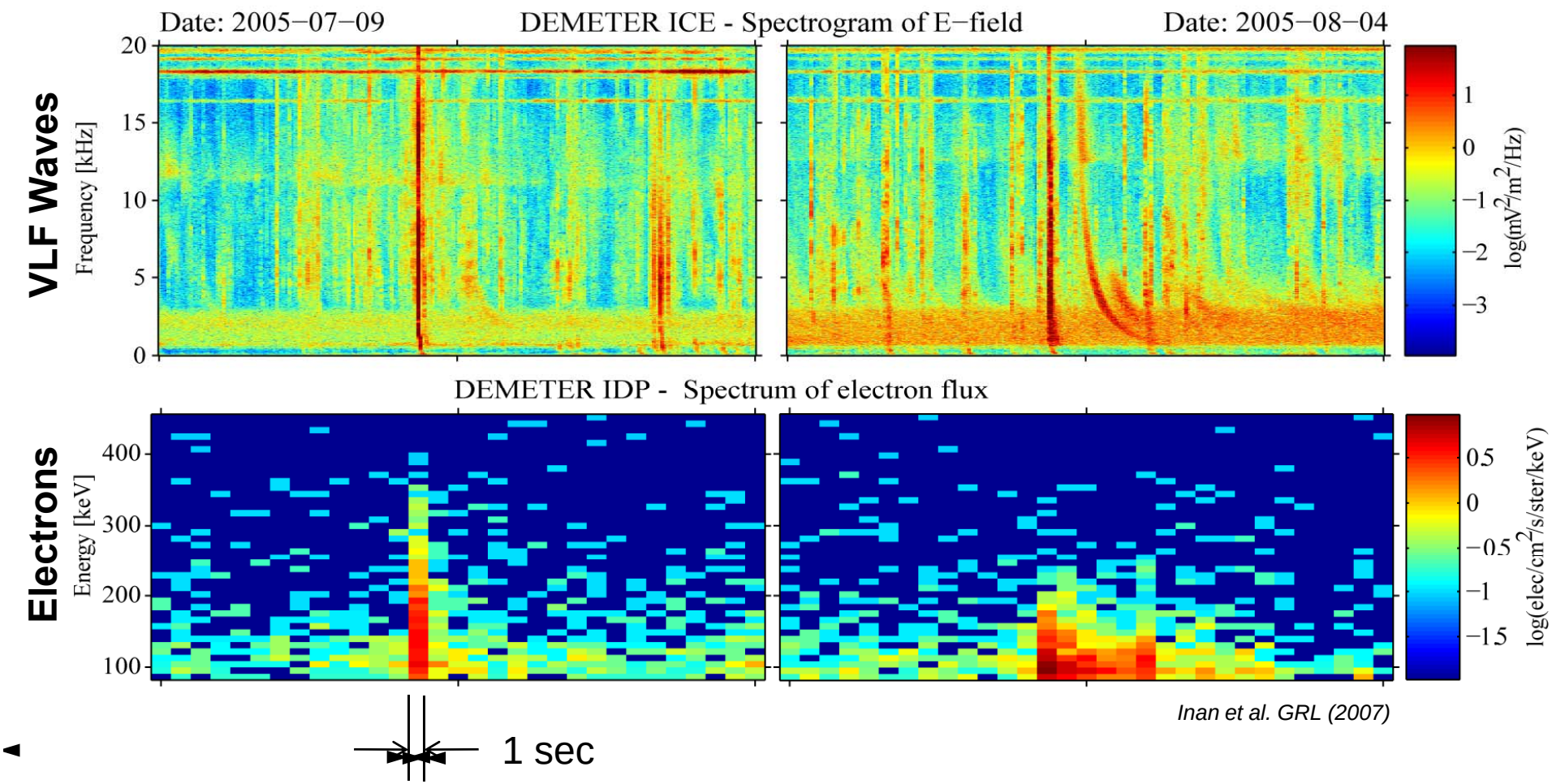


Summary

- DSX is manifest for launch as secondary payload on DMSP F-19 with launch in Oct 2012 (decision on launch date to be made in Jun 2011)
- DSX will make detailed measurements of in-situ VLF waves
  - Missing 20 dB of VLF power is a big inner magnetosphere question
- Tremendous opportunities for mono-static and bi-static VLF transmit-receive measurements
  - Determine VLF antenna transmission efficiency
  - Validate chorus – hiss conversion model
- Comprehensive particle detector suite will map poorly explored MEO region
  - Provides much needed data to update climatological radiation belt models used for spacecraft design
- Lots of good science to be done!

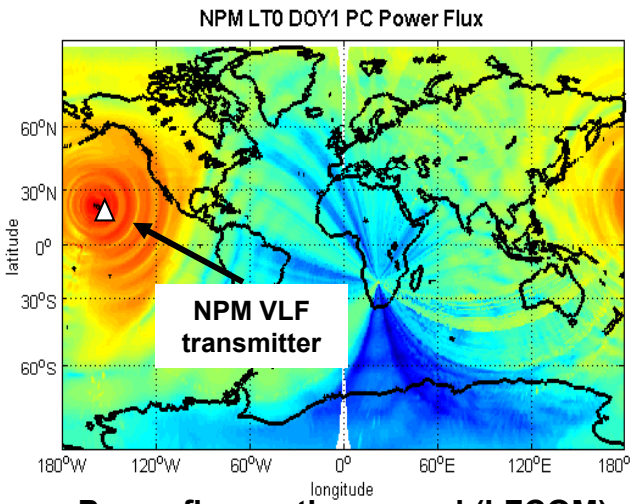
Backup Slides

Example LEP Events (DEMETER)

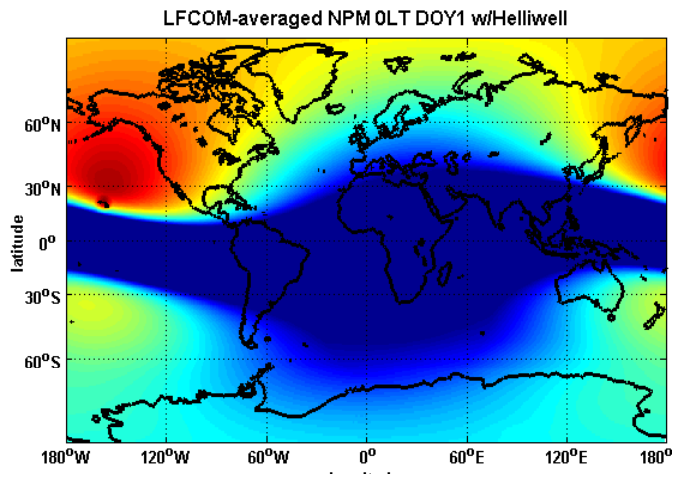


Inan et al. GRL (2007)

Ground-based VLF Injection

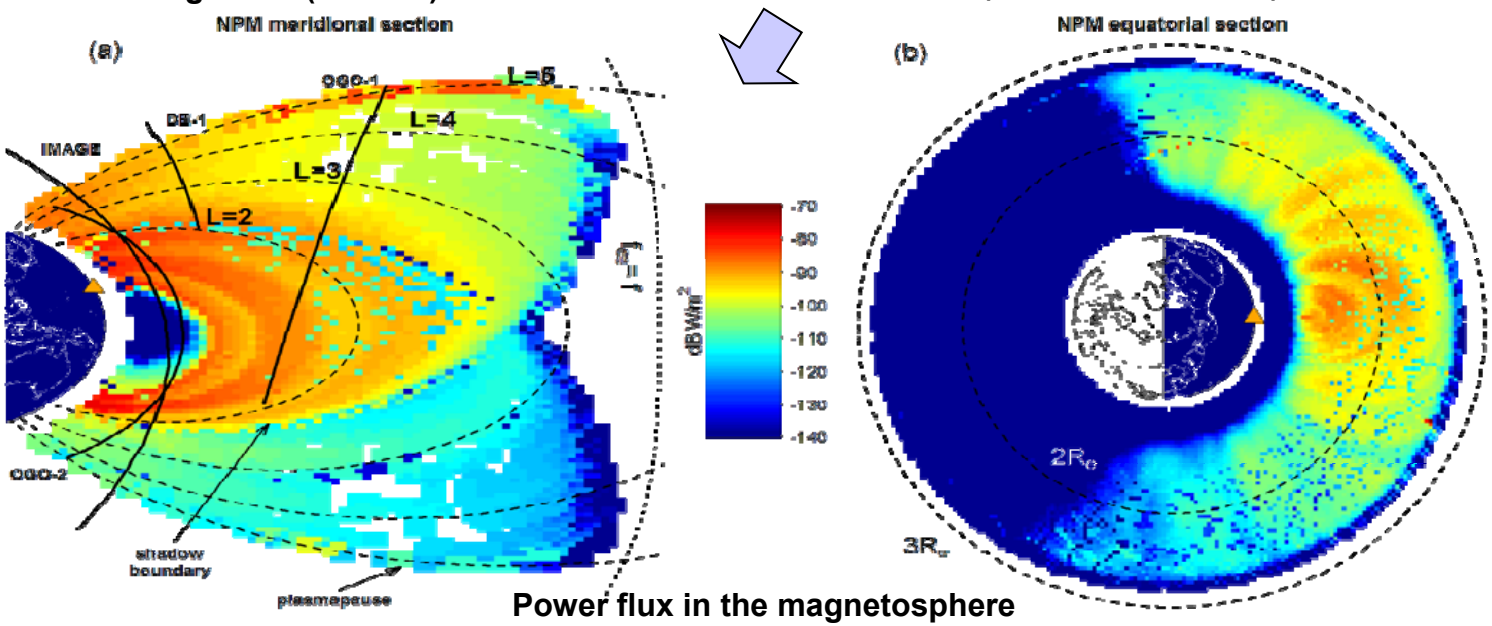


Power flux on the ground (LFCOM)



Power flux at 1000 km (LFCOM + Helliwell)

Sequence of "standard" models used to estimate VLF distribution in space

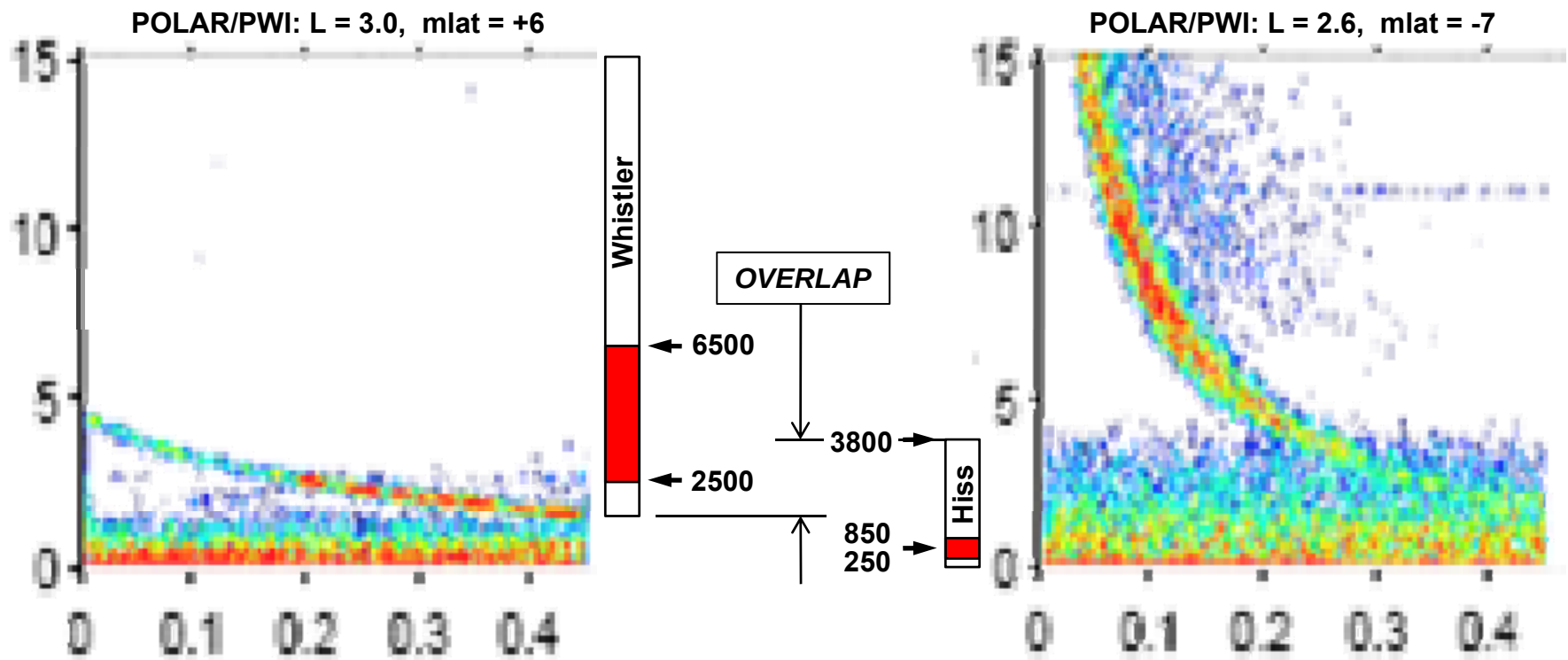


Power flux in the magnetosphere
(LFCOM + Helliwell + PowerTrace)

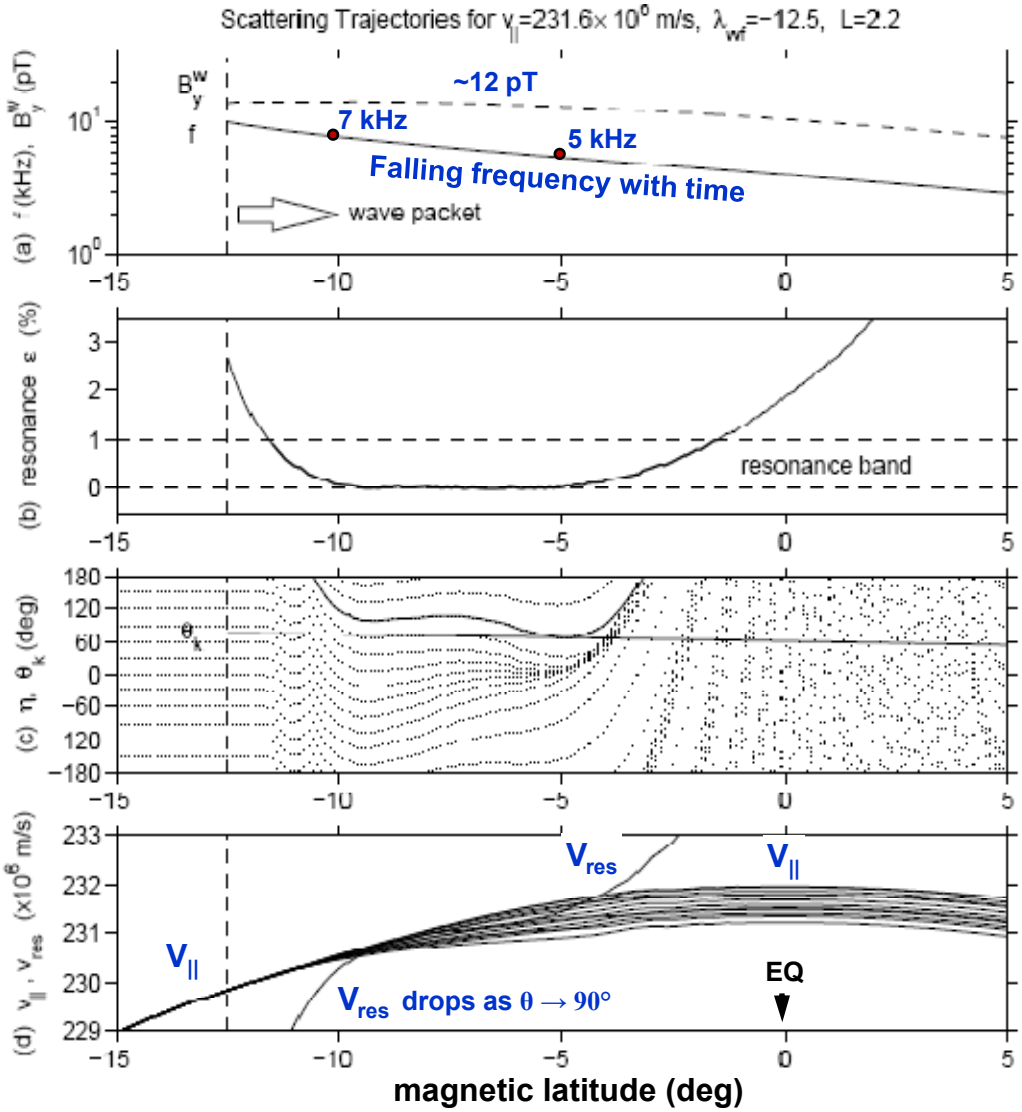
20dB? Could it be wave type?

Type of Wave	Plasmaspheric Hiss	Lightning-Generated Whistlers	VLF Transmitter Signals
Frequency/bandwidth, Hz	550 ± 300 Hz *(250 – 850 Hz)*	4.5 ± 2 kHz *(2.5 – 6.5 kHz)*	17.1 kHz ± 50 and 22.3 kHz ± 50 Hz
Propagation angle/width	45°± 22.5°	45°± 22.5°	45°± 22.5°
Wave intensity	10 pT	10 pT	variable
Occurrence rate	10% to 50%	3%	2.4% each

Abel and Thorne, "Electron scattering loss in Earth's inner magnetosphere: 1. Dominant physical processes," JGR, 103(A2), 2385– 2396



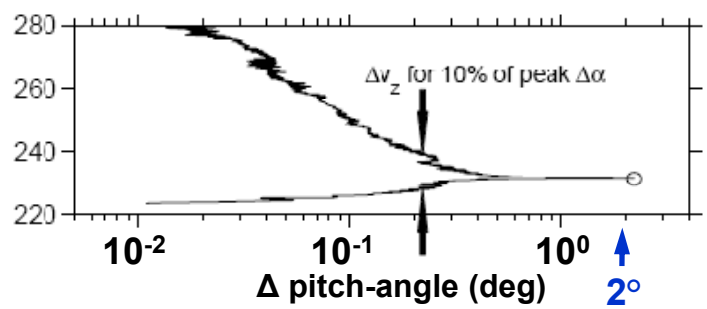
Whistler *f-t* Causes Δα Peaking



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 106, NO. A12, PAGES 29,745–29,770, DECEMBER 1, 2001

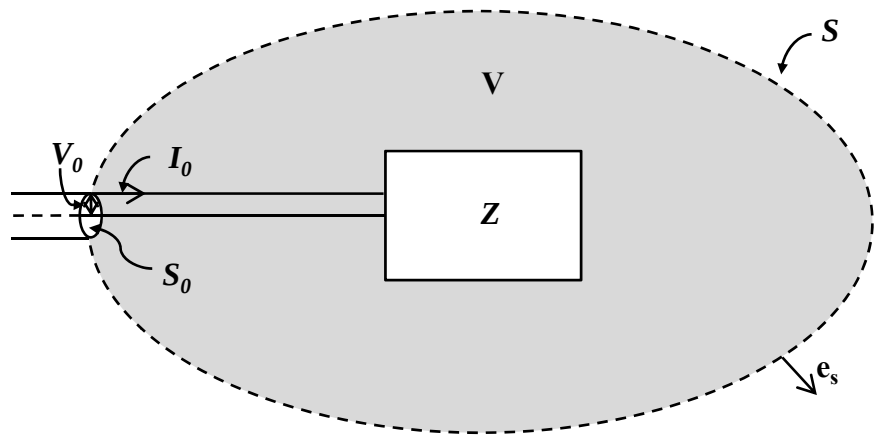
- **Natural f-t dispersion leads to extended resonance and order-of-magnitude greater scattering for same wave pT**
- **Extended resonance leads to particle phase-bunching and possible wave growth**
- **Calculated peak scattering for strong natural whistler could be as much as 1 deg**

Scattered Pitch-Angle vs. Vres for whistler with E_{100} = 50 V/m



Lauben (1998), Stanford Ph.D. Thesis

Antenna Basics



Fourier decomposition in time: $f(\mathbf{x},\mathbf{t}) = \mathrm{Re}\left[\hat{f}(\mathbf{x},\omega)\exp\{j\omega t\}\right]$

Poynting's Theorem:

$$\underbrace{P = \frac{1}{2}\hat{I}_0 * \hat{V}_0}_{\text{Time-averaged power into the system}} = \underbrace{\frac{1}{2} j\omega \int_V d^3x \left(\mu_0 \hat{\mathbf{H}}^* \cdot \hat{\mathbf{H}} - \varepsilon_0 \hat{\mathbf{E}}^* \cdot \hat{\mathbf{E}}\right)}_{\text{Energy stored in fields}} + \underbrace{\frac{1}{2}\int_V d^3x\, \hat{\mathbf{E}} \cdot \hat{\mathbf{J}}^*}_{\text{Ohmic losses}} + \underbrace{\frac{1}{2}\int_{S-S_0} d^2x\, \mathbf{e}_s \cdot \left(\hat{\mathbf{E}} \times \hat{\mathbf{H}}^*\right)}_{\text{Energy radiated out of system}}$$

Impedance: $Z = R + jX$

$$X = \omega L - \frac{1}{\omega C}$$

$$R = R_{ohmic} + R_{radiation}$$

Smaller

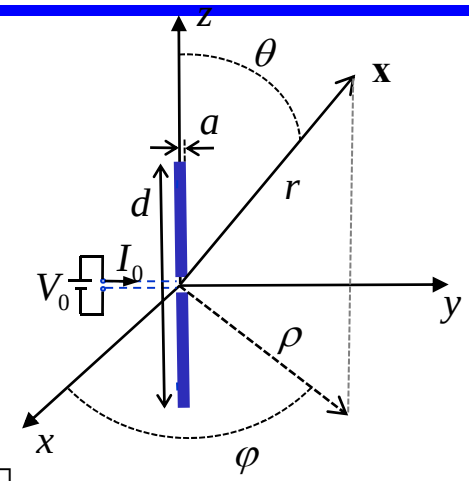
Bigger

Ohm's Law: $\hat{V}_0 = Z\hat{I}_0 \longrightarrow P = \frac{1}{2}|\hat{I}_0^{\,2}|Z$

$$\mathrm{P}_{rad} = \frac{1}{2}|\hat{V}_0^{\,2}|\frac{R_{rad}}{\left(R^2 + |X|^2\right)}$$

Linear Dipole in Cold Plasma

Solution to wave equation: $\tilde{\mathbf{E}} = j\omega\mu_0\left(\mathbf{kk} - k^2\mathbf{I} + \omega^2\mu_0\boldsymbol{\varepsilon}\right)^{-1}\cdot\tilde{\mathbf{J}}_{\mathbf{ext}}$.



$$\left(\mathbf{kk} - k^2\mathbf{I} + \omega^2\mu_0\boldsymbol{\varepsilon}\right)^{-1} = \frac{\boldsymbol{\Lambda}}{k_0^2\alpha\left(k^2 - k_+^2\right)\left(k^2 - k_-^2\right)},$$

$$\boldsymbol{\Lambda} = k^4\mathbf{nn} - k^2k_0^2\mathbf{L} + k_0^4\mathbf{W},$$

$$\alpha(\theta) = \bar{\varepsilon}_1\sin^2\theta + \bar{\varepsilon}_3\cos^2\theta,$$

$$\mathbf{L} = \begin{bmatrix} \bar{\varepsilon}_1\left(n_x^2 + n_y^2\right) + \bar{\varepsilon}_3\left(n_x^2 + n_z^2\right) & -j\bar{\varepsilon}_2\left(n_x^2 + n_y^2\right) + \bar{\varepsilon}_3 n_x n_y & \bar{\varepsilon}_1 n_x n_z - j\bar{\varepsilon}_2 n_y n_z \\ j\bar{\varepsilon}_2\left(n_x^2 + n_y^2\right) + \bar{\varepsilon}_3 n_x n_y & \bar{\varepsilon}_1\left(n_x^2 + n_y^2\right) + \bar{\varepsilon}_3\left(n_x^2 + n_z^2\right) & \bar{\varepsilon}_1 n_y n_z + j\bar{\varepsilon}_2 n_x n_z \\ \bar{\varepsilon}_1 n_x n_z + j\bar{\varepsilon}_2 n_y n_z & \bar{\varepsilon}_1 n_y n_z - j\bar{\varepsilon}_2 n_x n_z & \bar{\varepsilon}_1\left(1 + n_z^2\right) \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} \bar{\varepsilon}_1\bar{\varepsilon}_3 & -j\bar{\varepsilon}_2\bar{\varepsilon}_3 & 0 \\ j\bar{\varepsilon}_2\bar{\varepsilon}_3 & \bar{\varepsilon}_1\bar{\varepsilon}_3 & 0 \\ 0 & 0 & \bar{\varepsilon}_1^2 - \bar{\varepsilon}_2^2 \end{bmatrix}.$$

Electric field on external surface of antenna

Power: $$P = -\frac{1}{2}\int_V d^3x\,\hat{\mathbf{E}}\cdot\hat{\mathbf{J}}_{ext}^* = \boxed{-\frac{j\omega\mu_0}{16\pi^3k_0^2}\int_{-\infty}^{\infty} d^3k\,\frac{\tilde{\mathbf{J}}_{ext}^*\cdot\boldsymbol{\Lambda}\cdot\tilde{\mathbf{J}}_{ext}}{\alpha\left(k^2 - k_+^2\right)\left(k^2 - k_-^2\right)}}.$$

Surface current on antenna

Difficult integral requiring complex k-plane integration to preserve radiation boundary conditions

Radiation resistance: $R_{rad} = \dfrac{2\,\mathrm{Re}\{P\}}{\left|I_0\right|^2}$

Cold Plasma Basics

Maxwell's equations:

$$\nabla \cdot \varepsilon_0 \mathbf{E} = \rho,$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right),$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0,$$

Equation of motion for species *l*:

$$m_l \frac{d\mathbf{v}_l}{dt} = q_l \left(\mathbf{E} + \mathbf{v}_l \times \mathbf{B} \right) - \cancel{\upsilon_l \mathbf{v}_l},$$

Charge and current density:

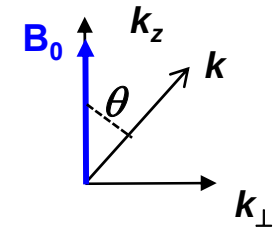
$$\rho = \sum_{l=1}^{N} n_l q_l,$$

$$\mathbf{J} = \sum_{l=1}^{N} n_l q_l \mathbf{v}_l.$$

Fourier representation:

$$f(\mathbf{x},\mathbf{t}) = \frac{1}{2}\left[\tilde{f}(\mathbf{k},\omega) \exp\{ j(\omega t - \mathbf{k}\cdot\mathbf{x}\} + c.c. \right],$$

Linear wave equation:



$$\left(\mathbf{kk} - k^2 \mathbf{I} + \omega^2 \mu_0 \boldsymbol{\varepsilon} \right) \cdot \tilde{\mathbf{E}} = j\omega \tilde{\mathbf{J}}_{\mathbf{ext}},$$

with dielectric tensor:

$$\boldsymbol{\varepsilon} = \varepsilon_0 \begin{bmatrix} \varepsilon_1 & j\varepsilon_2 & 0 \\ -j\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix},$$

$$\varepsilon_1 = 1 - \frac{\omega_{pe}^2 U_e}{\omega^2 - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \qquad \omega_{pi,e} = \sqrt{\frac{n_{i,e} q_{i,e}^2}{m_{i,e} \varepsilon_0}}$$

$$\varepsilon_2 = \frac{-\omega_{pe}^2 \omega_{ce}}{\omega\left(\omega^2 - \omega_{ce}^2\right)} + \frac{\omega_{pi}^2 \omega_{ci}}{\omega\left(\omega^2 - \omega_{ci}^2\right)}$$

$$\varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \qquad \omega_{ci,e} = \frac{\left| q_{i,e} \right| B_0}{m_{i,e}}$$

Normal modes defined by:

$$\left| \mathbf{kk} - k^2 \mathbf{I} + \omega^2 \mu_0 \boldsymbol{\varepsilon} \right| = A \left(\frac{ck}{\omega} \right)^4 - B \left(\frac{ck}{\omega} \right)^2 + C = 0,$$

with solutions:

$$\boxed{\left(\frac{ck_\pm}{\omega} \right)^2 = \frac{B \pm F}{2A},}$$

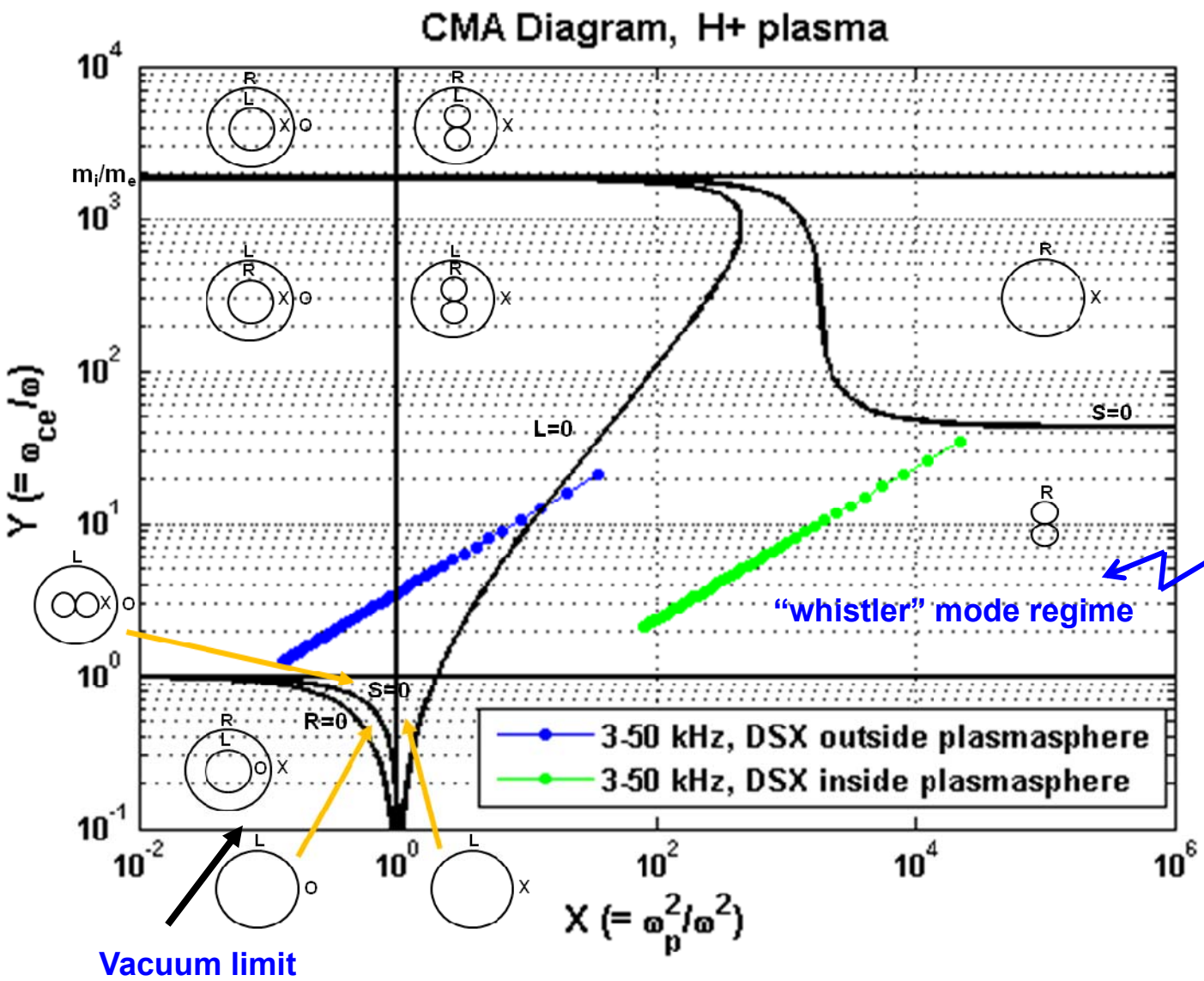
$$A = \varepsilon_1 \sin^2\theta + \varepsilon_3 \cos^2\theta,$$

$$B = \left(\varepsilon_1^2 - \varepsilon_2^2 \right) \sin^2\theta + \varepsilon_1 \varepsilon_3 (1 + \cos^2\theta),$$

$$C = \varepsilon_3 \left(\varepsilon_1^2 - \varepsilon_2^2 \right), \; F^2 = B^2 - 4AC \; > 0$$

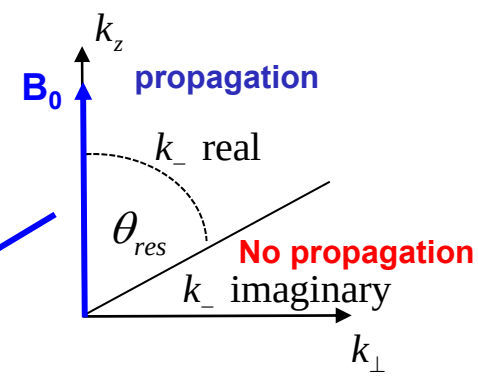
Lots of different notations : $\varepsilon_1, \varepsilon_2, \varepsilon_3 \Leftrightarrow S, D, P, R, L \Leftrightarrow X, Y$

Cold Plasma Mode Structure



Everything determined by:

$$\omega, \theta, \mathbf{B}_0, n_e = n_i$$



Resonance angle is when:

$\lambda \to 0$ ($k \to \infty$)

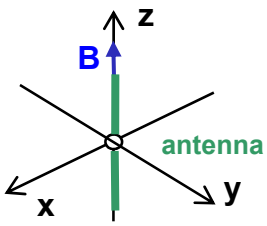
Defined by:

$$0 = \varepsilon_1 \sin^2 \theta_{res} + \varepsilon_3 \cos^2 \theta_{res}$$

Application to DSX

Assume a fixed linear current profile

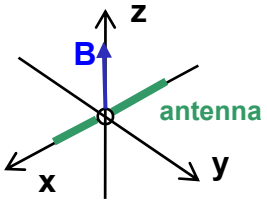
Parallel orientation:



Integrated up to $\theta_{cutoff} < \theta_{res}$

$$R_{rad} = -\frac{8\omega\mu_0}{\pi^2 k_0^2 d^2}\int_0^{2\pi} d\varphi \int_0^{\theta_c} d\theta \frac{\sin\theta}{\alpha\cos^4\theta}\frac{\Lambda_{33}(k_-)}{k_-^3\left(k_-^2 - k_+^2\right)}\times$$
$$J_0^2\left(k_- a\sin\theta\right)\sin^4\left(\frac{k_- d}{4}\cos\theta\right).$$

Perpendicular orientation:



$$R_{rad} = -\frac{8\omega\mu_0}{\pi^2 k_0^2 d^2}\int_0^{2\pi} d\varphi \int_0^{\theta_c} d\theta \frac{1}{\alpha\sin^3\theta\cos^4\varphi}\frac{\Lambda_{11}(k_-)}{k_-^3\left(k_-^2 - k_+^2\right)}\times$$
$$J_0^2\left(k_- a\left[1-\sin^2\theta\cos^2\varphi\right]^{1/2}\right)\sin^4\left(\frac{k_- d}{4}\sin\theta\cos\varphi\right).$$

DSX parameters:

Antenna length d [m]	80.0
Antenna radius a [m]	0.1
Frequency range ν [kHz]	3 - 50
Free-space wavelength λ_0 [m]	1.00E+05
Plasma density n_e [# cm^{-3}]	3.00E+03
Magnetic field B_0 [Gauss]	5.00E-02
(inside the plasmasphere)	

Choose cutoff wavelength:

$$k_{\max} = \frac{2\pi}{d/2} \rightarrow \eta_c^2 = \left(\frac{ck_{\max}}{\omega}\right)^2 \rightarrow \tan^2\theta_c = -\frac{P\left(\eta_c^2 - R\right)\left(\eta_c^2 - L\right)}{\left(S\eta_c^2 - RL\right)\left(\eta_c^2 - P\right)}$$

~ sheath size

Antenna Reactance

Electric dipole reactance is due to capacitance

Inside the plasmasphere

In vacuo: $$C_{ant} = \frac{\pi \varepsilon_0 d}{2\ln\left[d/2a\right]}$$

Static sheath (Mlodnosky & Garriott 1962):

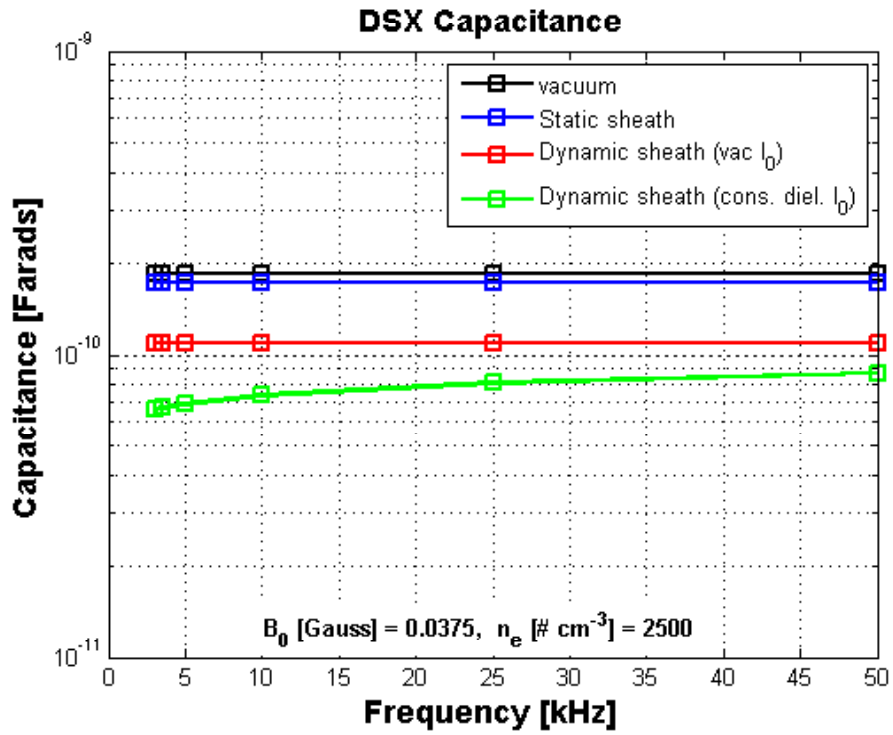
$$C_{sh} = \frac{\pi \varepsilon_0 d}{2\ln\left[r_{sh}/a\right]},$$

$$2\varepsilon_0 V_{sh} = -ner_{sh}^2 \ln\left[r_{sh}/a\right] + \frac{ne}{2}\left(r_{sh}^2 - a^2\right),$$

$$V_{sh} = \frac{kT_e}{e}\ln\left[\frac{v_{sat} + \sqrt{2/\pi}\, v_i}{v_e/2\pi}\right]$$

Dynamic sheath (Song, et al. 2007):

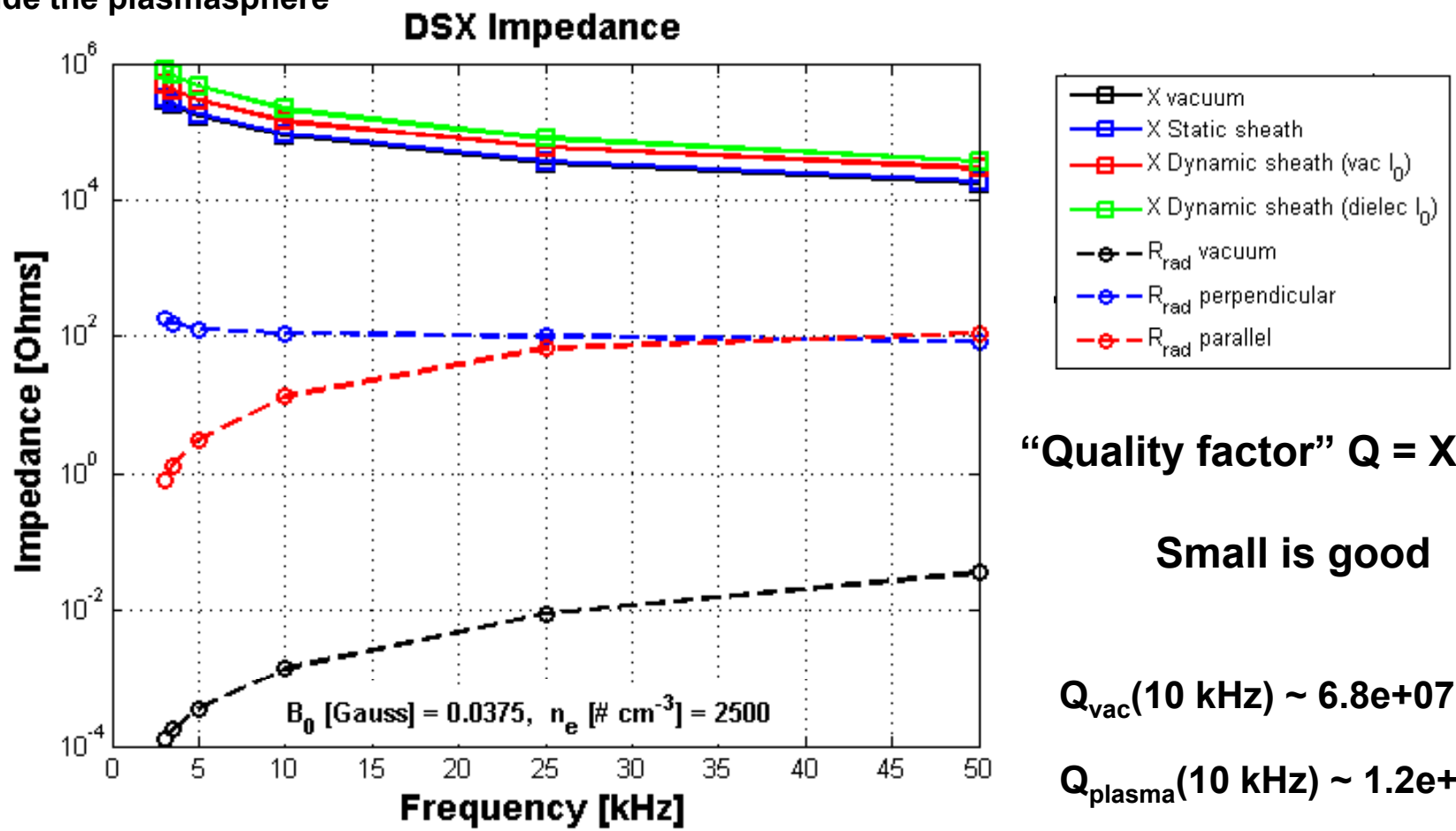
$$C_{sh} = \frac{\pi \varepsilon_0 d}{\ln\left[2I_0/\left(\pi ne\omega d a^2\right)\right]}$$



Probably not hugely different than in vacuo

Impedance Summary

Inside the plasmasphere



"Quality factor" $Q = X/R_{rad}$

Small is good

Q_{vac}(10 kHz) ~ 6.8e+07

Q_{plasma}(10 kHz) ~ 1.2e+03

Dipole Rrad in a plasma look better, at least in the linear limit…

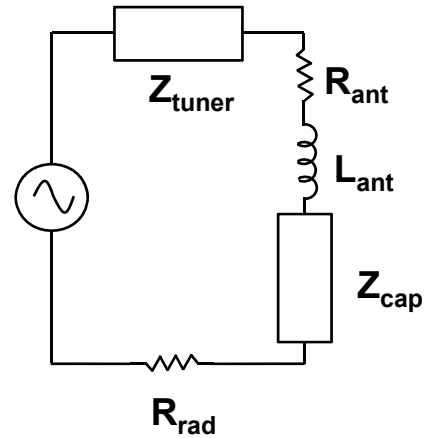
Plasma Antenna Model

***Vacuum* – dipole radiation *in vacuo* out to 10 km then cold plasma propagation**

***Linear cold plasma* – voltage and current distribution specified on antenna immersed in a cold plasma**

***Self-consistent linear cold plasma* – voltage on terminals specified, current distribution calculated self-consistently for antenna immersed in a cold plasma**

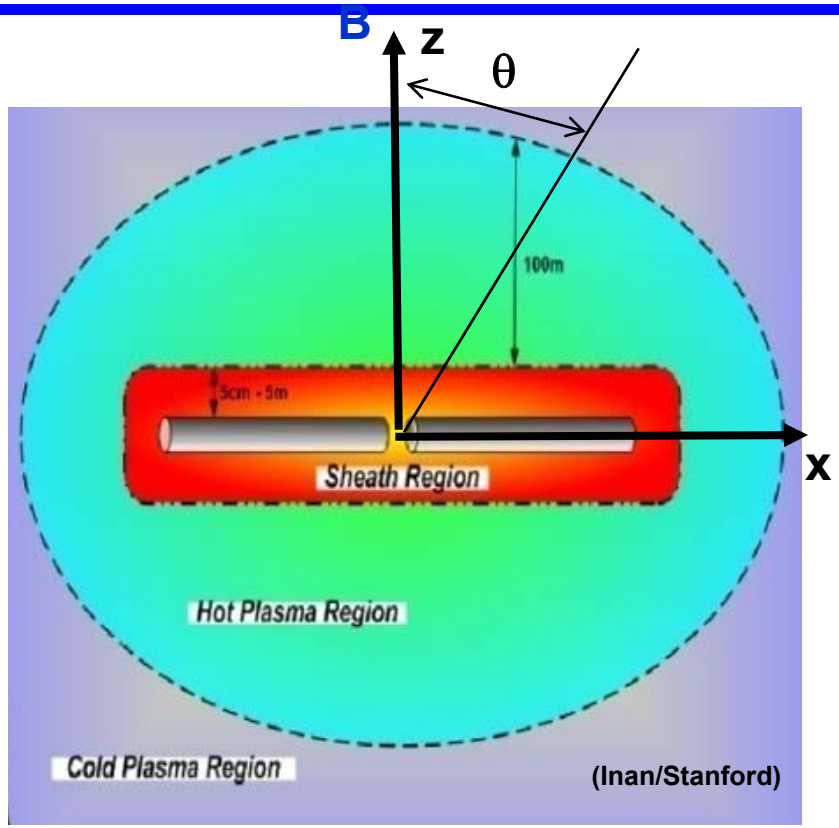
Sheath & plasma heating effects – determine sheath capacitance, resistance and antenna-sheath "effective" antenna current

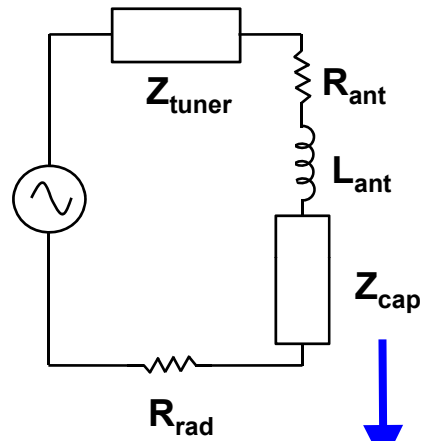


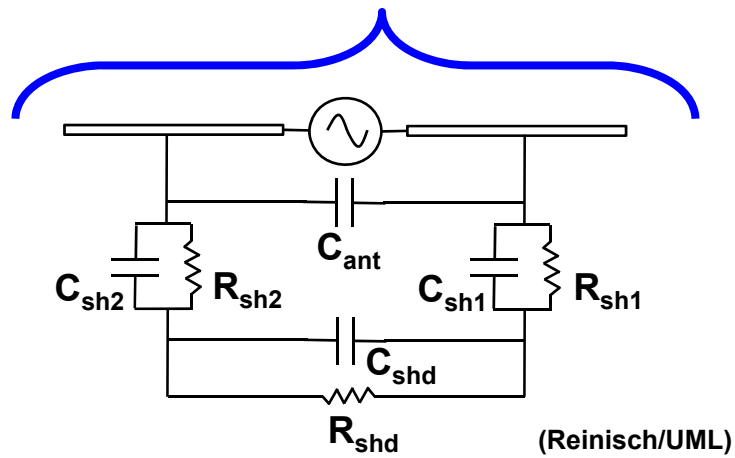
Effective circuit

Total power radiated as function of driving voltage

Maximum EfficiencyCircuits



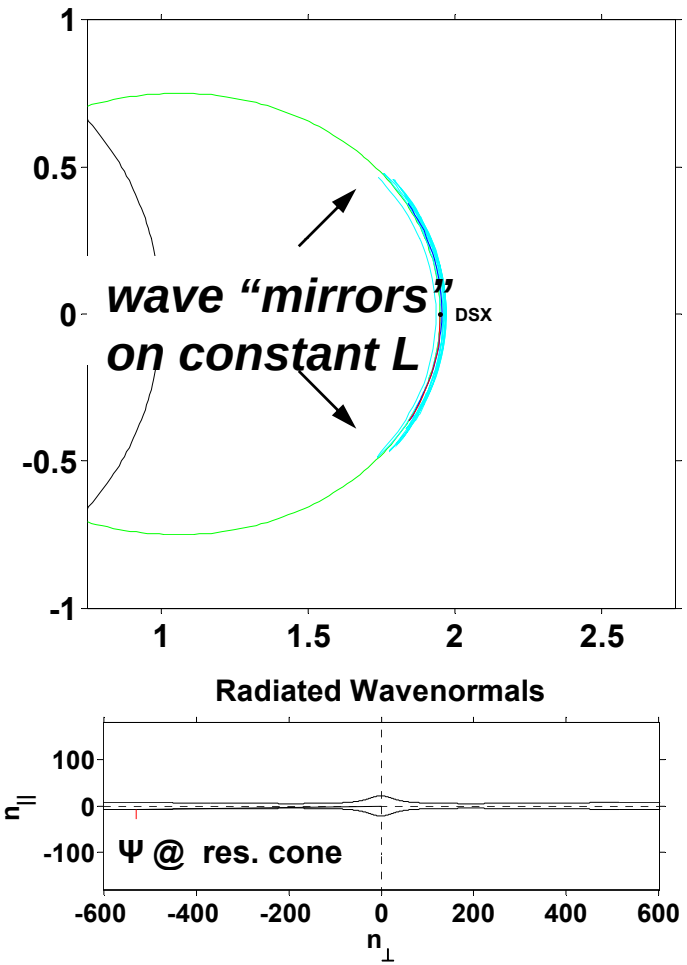




For an electric dipole: R_{ant}, Z_L << Z_{cap}

DSX Pattern Self-Measurement(2)

Lower-Hybrid Settling



Cavity Excitation