The Demonstrations & Science Experiment (DSX)
Update for RBSP Science Working Group

23-24 May 2011

Dave Lauben, Stanford
Gregory Ginet, MIT/LL
Michael Starks, AFRL
Mark Scherbarth, AFRL
Robert Helliwell, radioscience and magnetosphere expert, dead at 90

Robert Helliwell pioneered the study of how radio waves – both those naturally generated by lightning and manmade signals from a radio transmitter in Antarctica – interact with charged particles in the upper atmosphere.

BY MELISSAE FELLET

Late one night in 1950, a graduate student was monitoring radio waves emitted by distant lightning when strange descending whistling tones came from a speaker. The student, Jack Mallinckrodt, mentioned the experience to his adviser, Robert Helliwell.

"I suggested that if he took a short vacation perhaps the sounds would go away," Helliwell wrote in an article for the October 1982 issue of Stanford Engineer. "But he didn't and they didn't. My curiosity was finally aroused and I spent a late night with Jack at the receiving station. Luckily, we both heard two distinct whistles and I was instantly converted to belief in the reality of a strange new phenomenon."

This chance observation started Helliwell on decades of research that led him from Stanford to Antarctica as he followed these mysterious radio noises and later sought to reproduce them with a transmitter. Through this work, he made fundamental discoveries about how radio waves can be used to investigate the ionized atmosphere high above Earth's surface.

"He was a pioneer," said Don Carpenter, professor emeritus of electrical engineering and one of Helliwell's colleagues. "He did some of the earliest observations and interpretations of phenomena in our field."

Helliwell died on May 3 in Palo Alto of complications from dementia. He was 90. A service will be held at Stanford Memorial Church on June 7 at 3 p.m.
Outline

- Introduction and Motivation
- DSX spacecraft and experimental payloads
- Science question – where is the 20 dB?
- Science question – radiation pattern in plasma?
- Conjunction opportunities
- Summary
Wave-Particle Interactions

Particles mirroring below 100 km are “lost”

Electromagnetic waves in the Very Low Frequency (VLF) range (3-30 kHz) scatter and accelerate radiation belt electrons through cyclotron resonance interactions.

L shell = distance/RE

Particles pitch-angle

Electromagnetic waves

Waves from CRRES (1990)
Particle lifetime along field lines
(approximate 1D solution)

\[
\frac{\partial f}{\partial t} = \frac{1}{T \sin \alpha_0 \cos \alpha_0} \frac{\partial}{\partial \alpha_0} \left[ D_{\alpha_0} T \sin \alpha_0 \cos \alpha_0 \frac{\partial f}{\partial \alpha_0} \right]
\]

Full 3D global, time dependent particle distributions
\[ X_i = (L, E, \alpha) \]

Quantitative understanding of VLF wave power distribution & resultant wave-particle interactions is crucial for radiation belt specification & forecasting.

Wave-particle resonance condition
\[
\omega - k || v || = -n \frac{\Omega_e}{\gamma}
\]

Diffusion coefficients = sum over resonances
\[
D_{\alpha_0} = \pi \sum_n \cos^7 \lambda \frac{\Omega_e^2}{\eta ||} \left( \frac{\partial \omega}{\partial \lambda} \right)^{-1} \frac{B_w^2}{B^2} \frac{\dot{j}_n \Phi_n(\theta)}{\gamma^2(v/c)}
\]

\[ \dot{j}_n = \begin{cases} 1, & n \neq 0, \\ \sin^4 \alpha, & n = 0 \end{cases} \]
Non-Local Origins of Hiss

- Whistler-mode hiss helps control the lifetime of radiation belt electrons.
- Hiss may originate in chorus waves, and enter the plasmasphere via high latitude ray paths.

Conjunction experiments with space-based VLF transmitters and receivers can definitively probe the nature of ray-paths.
Map the MEO Environment

Satellite designers need a definitive model of the trapped energetic particle and plasma environment to include:

- Quantitative accuracy
- Indications of uncertainty
- Flux probability of occurrence and worst cases for different exposure periods
- Broad energy ranges
- Complete spatial coverage

MEO is sorely under sampled!
New Radiation Belt Model AP9/AE9

- New AP-9/AE-9 model being developed by NRO - AFRL - Aerospace – MIT/LL - LANL consortium
- Provides significant improvement in spectral coverage, error estimation and statistical output
- Needed by satellite engineers to control risk, maximize capability and reduce cost
- Version Beta released Apr 2010 and now being evaluated by 20+ independent spacecraft engineers from industry and government – Version 1.0 due in June 2011
- Version 2.0 (~2015) will utilize measurements from NASA Radiation Belt Storm Probes (RBSP) and AFRL DSX missions
## Past Active VLF Space Experiments

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Orbit</th>
<th>Epoch</th>
<th>TX (RX) Freq. [kHz]</th>
<th>TX Type</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOFTI I</td>
<td>166 x 960 km 28.4°</td>
<td>21 Feb 61 – 30 Mar 61</td>
<td>(18)</td>
<td>-</td>
<td>First space VLF experiment designed to measure ground stations (NBA, Panama).</td>
</tr>
<tr>
<td>LOFTI - IIA</td>
<td>170 x 925 km 70°</td>
<td>15 Jun 63 – 15 Jul 63</td>
<td>(10-18)</td>
<td>-</td>
<td>Focus was on impedance variations as functions of environment.</td>
</tr>
<tr>
<td>OV3-3</td>
<td>362 x 4488 km 81.6°</td>
<td>4 Aug-66 – 30 May 67</td>
<td>Broadband</td>
<td>Solar cell current noise</td>
<td>Focus on plasma density influences on impedance variation.</td>
</tr>
<tr>
<td>OV1-21S (1971-67A)</td>
<td>800 x 921 km 87.6°</td>
<td>7 Aug 71 – 3 Sep 71</td>
<td>0.4 – 14.5</td>
<td>Electric dipole: 2 x 16 m elements, Voltage: 10 mV - 100 V peak-to-peak</td>
<td>Observed impedences are in reasonable agreement with linear plasma sheath dominated predictions. No far-field measurements.</td>
</tr>
<tr>
<td>OV1-20S (1971-67B)</td>
<td>75 x 1948 km 92°</td>
<td>7 Aug 71 – 28 Aug 71</td>
<td>300</td>
<td>Electric dipole: 2 x 1.27 cm elements, 2.45 cm separation Voltage: 30 V</td>
<td>Observed resonance cone and angle/width compared favorably to cold and warm plasma theory predictions.</td>
</tr>
<tr>
<td>Activny</td>
<td>500 x 2500 km 82.6°</td>
<td>28 Sep 88 – 30 Apr 90</td>
<td>9.5 – 10</td>
<td>Magnetic loop: 20 m diameter, 1 m thick</td>
<td>Antenna did not deploy properly. Measurements by DE-1 indicate &lt; 10 mW radiated power (no detections.)</td>
</tr>
<tr>
<td>*IMAGE</td>
<td>1000 x 45922 km 40°, precesses</td>
<td>25 Mar 00 - 18 Dec 05</td>
<td>3 - 3000</td>
<td>Electric dipole: X: 125 m - 250 m Power: 0.1 mW -10 W</td>
<td>Not optimized for VLF transmission but good sheath impedance measurements.</td>
</tr>
<tr>
<td>DSX</td>
<td>6000 x 12000 km 120° retrograde</td>
<td>Oct 2012 (pending)</td>
<td>3 – 50000</td>
<td>Electric dipole: Y: 80 m tip-to-tip</td>
<td></td>
</tr>
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</table>

*Heritage for DSX*
Three science experiments:

1) **Wave-particle interactions (WPIx)**
   - Determine efficiency of injecting VLF into space plasmas *in situ*
   - Determine global distribution of natural & man-made ELF-VLF waves
   - Characterize and quantify wave-particle interactions

2) **Space weather (SWx)**
   - Map MEO radiation & plasma environment
   - Diagnose in-situ environment for wave generation experiments

3) **Space environment effects (SFx)**
   - Quantify effects of MEO environment on new technologies
   - Determine physical mechanisms responsible for material breakdown
**DSX Experimental Apparatus**

**Wave-Particle Interactions (WPIx)**
- VLF transmitter & receivers
- Loss cone imager
- Vector magnetometer

**Space Weather (SWx)**
- 5 particle & plasma detectors

**Space Environmental Effects (SFx)**
- NASA Space Environment Testbed
- AFRL effects experiment

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**AC Magnetometer (GSFC)**
- Tri-axial search coils

**VLF Transmitter & Receivers**
- Broadband receiver (Stanford)
- Transmitter & tuning unit (UML)

**Z-Axis Booms**
- VLF E-field Rx

**Y-Axis Booms**
- VLF E-field Tx/Rx

**Loss Cone Imager (BU)**
- High Sensitivity Telescope
- Fixed Sensor Head

**8 m**

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**DC Vector Magnetometer**

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**DSX satellite**

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**Loss Cone Imager (BU)**

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**Boom deployment test**

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**DSX being integrated!**

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**ESPA Ring**
- Interfaces between EELV & satellite
Wave-Particle Interactions Payload (WPIx)

- **Receiver** (Stanford, Lockheed-Martin, NASA/Goddard):
  - Three search coil magnetometers (3 B components)
  - Two dipole antennas (2 E components)
  - Frequency range: 100 – 50 kHz
  - Sensitivity 1.0e-16 V²/m²/Hz (E) & 1.0e-11 nT²/Hz (B)

- **Transmitter** (UMass Lowell, SWRI, AFRL):
  - 3 – 50 kHz at up to 5 kV (9 kV at end of life)
  - 50 – 3000 kHz at 1W (local electron density)

- **Loss Cone Imager** (Boston University, AFRL)
  - High Sensitivity Telescope (HST): measures 100 – 500 keV e- with 0.1 cm²-str geometric factor within 6.5 deg of loss cone
  - Fixed Sensor Heads (FSH): 130 deg x 10 deg of pitch angle distribution for 50 – 700 keV electrons every 167 msec

- **Vector Magnetometer** (UCLA, UMich)
  - 0 – 8 Hz three-axis measurement at ±0.1 nT accuracy
Space Weather Payload (SWx)

Comprehensive SWx sensor suite will map full range of MEO space particle hazards

CEASE - Compact Environment Anomaly Sensor (Amptek, AFRL)
LEESA - Low Energy Electrostatic Analyzer (AFRL)
LIPS - Low Energy Imaging Particle Spectrometer (PSI)
HIPS - High Energy Imaging Particle Spectrometer (PSI)
HEPS - High Energy Particle Sensor (Amptek, ATC)
Space Weather Effects Payload (SFx)

NASA Space Environment Testbed (SET)

- Correlative Environment Monitor (QinetiQ)
  - Dosimeter & deep-dielectric charging package
- DIME (Clemson Univ)
  - Dosimetry Intercomparison and Miniaturization
- ELDRS (Arizona State)
  - Development of space-based test platform for the characterization of proton effects and Enhanced Low Dose Rate Sensitivity (ELDRS) in bipolar junction transistors
- COTS-2 (CNES and NASA)
  - Validation of single event effects mitigation via fault tolerant methodology

SFx experiments will quantify MEO environment effects on advanced spacecraft technologies & determine basic physics of breakdown

AFRL/PRS “COTS” sensors
Objective: directly measure changes in
- Optical transmission,
- Thermal absorption
- Thermal emission
due to MEO radiation environment
Where is the 20 dB?

Abel & Thorne (1998)

Ground transmitter VLF power needed in the inner magnetosphere... but where is it?

Starks, et al. (2008)
20dB? It’s not the Absorption Model…

- The four models operate entirely differently: empirical, mode theory, finite differences, full wave

- All predict essentially the same ionospheric penetration fields

- However, all of them overestimate the fields by 20 dB or more.

Questions:
- Where is the transmitter power going?
  - Non-linear lower-hybrid wave – density fluctuation scattering?
- What is scattering the particles at L < 2?
  - Could lightning be more effective than previously thought?
Whistler range 6.5 – 12 kHz occurs frequently, yet not presently modeled. Impacts same 500 keV electrons at L=2 but with m=1 order instead of m=2.

### Table 1. Adopted Wave Parameters, in Abel & Thorne (1998a)

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<td>10% to 50%</td>
<td>3%</td>
<td>2.4% each</td>
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\[ \omega - k \parallel \parallel v \parallel = -m \Omega / \gamma \]

Abel & Thorne (1998)

Largest Impact @ L=2

L=2, 500 keV

2.5 – 6.5 kHz Whistler Model

Important?
Typical Whistlers in the Slot Region

POLAR/PWI: Example waves in equatorial slot region
Lightning Magnetic Field Line Footprints

Lightning Flash Rates at DSX Footprints

Lightning Flash Rates Jul

Lightning Flash Rates Dec

Mission day [day 1 = 24-Nov-2012 06:00:00]
DSX provides ample Lightning and Ground Tx overflights for comparison
Radiation Pattern: Linear Dipole in Vacuo

**Potentials:**
\[ \mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A} \]

**Lorentz gauge:**
\[ \nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0 \]

**Green’s function:**
\[ \mathbf{A}(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \int d^3x' \int_{-\infty}^{t} dt' \frac{\mathbf{J}(\mathbf{x}, t)}{|\mathbf{x} - \mathbf{x}'|} \delta \left( t' - t + \frac{\mathbf{x} - \mathbf{x}'}{c} \right) \]

**Boundary conditions:**
\[ \hat{E}_z(\rho = a) = 0, \quad 0 < |z| < \frac{d}{2} \]
\[ \hat{E}_z(\rho = a) = -\hat{V}_0 \delta(z), \quad z = 0 \]

**Hallén integral equation:**
\[ \left( \frac{\partial^2}{\partial z^2} + k_0^2 \right) \int_{-d/2}^{d/2} dz' I(z', \rho) \frac{\exp\{-jk_0|z'-r|\}}{r'} = -\frac{4\pi jk_0^2}{\omega} \hat{V}_0 \delta(z) \]

**Can be solved analytically in electrically thin & short limit:**
\[ d \ll \lambda_0, \quad a \ll \lambda_0, \quad \lambda = \frac{2\pi}{k_0} \]
\[ I_0 = \frac{j\pi V_0 k_0 d}{2Z_0 \left[ \ln \left( \frac{d}{2a} \right) \right]^{-1}}. \]
Antenna Radiation in Plasma

- Far-field power radiated \( \sim \frac{d}{dt}(I_1 + I_2 + I_{plasma}) \)
- Sheaths can change antenna reactance
- Antenna fields heat local plasma
- Anisotropic medium can dictate complicated far-field radiation pattern
Radiation in Plasma: Resonance Cones

In the laboratory


In space

**Plasma Environment**

**Magnetic field**

\[ B \text{ (Gauss)} \]

**Plasma density**

\[ n \text{ (#/cm}^3) \]

**Characteristic frequencies**

Stix S, D, P, R, L → \( R_{rad}(k_x, k_y, k_z) \)
DSX Radiation Patterns

Inside the plasmasphere

Parallel

Perpendicular

θ_{cutoff} = 89.4° – 68.3°, ν = 3 – 50 kHz

Magnetoplasma Strongly Controls Far-Field Whistler-Mode Radiation Pattern
DSX Pattern Self-Measurement

Atmospheric Reflection

*topside ionosphere reflection point*

return waves **straddle DSX (blue)**

Boomerang Return

*magnetospheric reflection point*

return waves **straddle DSX (blue)**

Radiated Wavenormals

$\Psi \sim -62$ deg

Radiated Wavenormals

$\Psi \sim 88$ deg
What Resonant Energies?

Evaluate $\omega - k_|| v_|| = -m \Omega / \gamma$ along ray paths to solve for $v_||$

$L = 2.1 \quad Preliminary! \quad L = 2.6$

DSX @ (L = 2.1, $\lambda = 15.0$ deg), $f = 3.0$ kHz, $\psi_0 = 61.19650$

DSX @ (L = 2.6, $\lambda = 15.0$ deg), $f = 3.0$ kHz, $\psi_0 = 84.43180$
- But, whistler ray paths are NOT LINE OF SIGHT!
- They're not generally field-aligned, either!

- This animation shows a 0.1 sec pulse of $f = 6$ kHz rays for duration 2.0 sec
- Rays launched w/in 3 deg of local resonance cone, at $L = 2.1$, $mlat = 15$ deg
- The 1$^{st}$ pass rays = red, 2$^{nd}$ = blue, 3$^{rd}$ = cyan
DSX Far-Field Propagation(2)

t = 0.59 s

Significant rays to alt < 700 km

Significant size wave packets sweep L < 2

f = 6.0 kHz, L = 2.1, \lambda = 15.0 : pw = 0.100 s, t = 0.590 s
Extending to 3D Propagation…

Wave energy settles (and damps) in longitudinal “wings”
Morphology is frequency and wavenormal dependent

(a) $f = 1000 \text{ Hz } \theta = 35^\circ$
(b) $f = 1000 \text{ Hz } \theta = 75^\circ$
(c) $f = 3000 \text{ Hz } \theta = 35^\circ$
(d) $f = 3000 \text{ Hz } \theta = 75^\circ$
Critical Unknown:
Importance of scattering and mode-conversion on power and k-spectrum
Satellite at 6000 km altitude, 0° magnetic lat, vacuum antenna limit; 10 kHz

Magnetospheric reflection destines rays for lower altitudes at 10 kHz as compared to 3 kHz
Magnetospheric reflection is not a factor above about 12 kHz. Transmitter energy is concentrated around the field line of the transmitter.
At high altitude, 30 kHz does not propagate well, losing intensity extremely quickly.
Satellite at 6000 km altitude, 30° magnetic lat, vacuum antenna limit; 10 kHz

The off-equatorial transmitters lead to very complex field distributions.
Joint Experiment Opportunities – Space

- **Cassiope/Enhanced Polar Outflow Probe (E-PoP), CSA, CRC (James), NRL (Siefring, Bernhardt)**
  - 300 x 1500 km, polar inclination, launch 2011 (?)
  - Radio Receiver Instrument (RRI), ELF-VLF 10 Hz -30 kHz, two-axis E-field
  - Fast Auroral Imager (FFI), ~ 1 MeV electrons

- **Radiation Belt Storm Probes (RBSP), NASA**
  - 2 satellites in GTO, < 18 deg incl, launch no earlier than fall 2011
  - Electric and Magnetic Field Instrument Suite and Integrated Science Suite (EMFISIS, Univ. of Iowa, Kletzing), 3 axis B-field, 2 axis E-field 10 Hz – 12 kHz (1 channel E-field 10 kHz – 400 kHz)
  - Magnetic Electron-Ion Spectrometer (MagEIS, BU & Aerospace, Spence & Blake), 40 keV – 10 MeV electrons
  - Relativistic Electron-Proton Telescope (REPT, BU & Univ. of Colorado, Spence & Baker), 2 MeV – 10 MeV electrons
  - RBSP Ion Composition Explorer (RBSPICE, NJIT, Lanzerotti), 25 keV – 500 keV electrons

- **TARANIS, CNES, Stanford Co-PI (Inan), follow on to DEMETER**
  - 700 km, polar, launch 2011(?)
  - IMM-MF, B-field 3 component, ~2 Hz – 20 kHz, 1 component 10 kHz – 1MHz
  - IDEE, electron detectors, 70 keV – 4 MeV

- **VPM, AFRL (Starks), Stanford (Linscott)**
  - Cubesat in ~700 km, high-inclination LEO
  - VLF receiver & loss-cone electron detector
  - Approved by AFRL in Feb 2011 for development & launch in DSX timeframe

- **ORBITALS, CSA, Univ. of Calgary (Mann), Univ. of Colorado (Baker)**
  - GTO, launch (?)
  - SCM, B-field up to 20 kHz
  - EPS, electrons 25 keV – 12 MeV
Active Injection: Test Chorus → Hiss

DSX Can Inject “Just Above” the Chorus → Hiss Entry Points

[Image: Diagram showing the injection of DSX f = 3kHz and DSX f = 6kHz into the chorus and hiss regions, labeled with Night side, 30°, 20°, 10°, and 0°.]

[Bortnik et al., Nature 452, 6 March 2008]
DSX – RBSP Conjunctions

Magnetic footprints

1 week (typical)

Closest approach ~ 422 km
DSX – RBSP Conjunction Occurrence

DSX - RBSP $L^*$ Conjunctions, $N_{\text{con}} = 2436$

DSX - RBSP $L^* - MLT$ Conjunctions, $N_{\text{con}} = 127$

DSX - RBSP $L^* - MLT - K$ Conjunctions, $N_{\text{con}} = 23$

L shell
Magnetic field line
Magnetic point
DSX – LEO (Demeter-like) Conjunctions

Magnetic footprints

DSX  DEMETER
24-Nov-2012 06:00:00 - 01-Dec-2012 05:55:00 (5 min resolution)

1 week (typical)

Closest approach ~ 5393 km
DSX – LEO Conjunction Occurrence

DSX-DEMETER L⁺ Conjunctions, \(N_{\text{conj}} = 2003\)

DSX-DEMETER L⁺ - MLT Conjunctions, \(N_{\text{conj}} = 58\)

None

L shell

Magnetic field line

Magnetic point
Joint Experiment Opportunities – Ground

• High-Frequency Active Auroral Research Program (HAARP, AFRL)
  – Electrojet-modulated VLF antenna at L ~ 4.8 with extensive frequency & mode control

• DoD VLF transmitters, TIPER program (AFRL & Stanford)
  – Keyed transmissions from NWC at Churchill, Australia, L ~ 1.3, 19.8 kHz, 1 MW
  – Mobile VLF transmitter broadcasts, ~ 18 kHz

• Balloon Array for Radiation-belt Relativistic Electron Losses (BARREL)
  – Measurement of precipitating MeV electrons at high latitudes with 5-8 balloon flotilla aloft for ~ one month
Summary

• DSX is manifest for launch as secondary payload on DMSP F-19 with launch in Oct 2012 (decision on launch date to be made in Jun 2011)
• DSX will make detailed measurements of in-situ VLF waves
  – Missing 20 dB of VLF power is a big inner magnetosphere question
• Tremendous opportunities for mono-static and bi-static VLF transmit-receive measurements
  – Determine VLF antenna transmission efficiency
  – Validate chorus – hiss conversion model
• Comprehensive particle detector suite will map poorly explored MEO region
  – Provides much needed data to update climatological radiation belt models used for spacecraft design
• Lots of good science to be done!
Backup Slides
Example LEP Events (DEMETER)

DEMETER ICE - Spectrogram of E-field

DEMETER IDP - Spectrum of electron flux

Date: 2005–07–09

Date: 2005–08–04

VLF Waves

Frequency [kHz]

Energy [keV]

Electrons

Inan et al. GRL (2007)
Ground-based VLF Injection

Sequence of “standard” models used to estimate VLF distribution in space
**20dB? Could it be wave type?**

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**POLAR/PWI:** L = 3.0, mlat = +6

**POLAR/PWI:** L = 2.6, mlat = -7
Whistler $f$-$t$ Causes $\Delta \alpha$ Peaking

- Natural $f$-$t$ dispersion leads to extended resonance and order-of-magnitude greater scattering for same wave pT
- Extended resonance leads to particle phase-bunching and possible wave growth
- Calculated peak scattering for strong natural whistler could be as much as 1 deg

Lauben (1998), Stanford Ph.D. Thesis
Antenna Basics

Fourier decomposition in time: \[ f(x, t) = \text{Re} \left[ \hat{f}(x, \omega) \exp \{ j \omega t \} \right] \]

Poynting’s Theorem:
\[
P = \frac{1}{2} \hat{I}_0 \hat{V}_0 = \frac{j \omega}{2} \int V d^3x \left( \mu_0 \hat{H}^* \cdot \hat{H} - \varepsilon_0 \hat{E}^* \cdot \hat{E} \right) + \frac{1}{2} \int d^3x \left( \hat{E} \cdot \hat{J}^* + \frac{1}{2} \int_{S-S_0} d^2x \left( \hat{E} \times \hat{H}^* \right) \right)
\]

Time-averaged power into the system
Energy stored in fields
Ohmic losses
Energy radiated out of system

Impedance: \[ Z = R + jX \]

Ohm’s Law:
\[ \hat{V}_0 = Z \hat{I}_0 \rightarrow P = \frac{1}{2} |\hat{I}_0|^2 |Z| \]

\[ X = \omega L - \frac{1}{\omega C} \]

\[ R = R_{\text{ohmic}} + R_{\text{radiation}} \]

Time-averaged power into the system
Energy stored in fields
Ohmic losses
Energy radiated out of system

\[ P_{\text{rad}} = \frac{1}{2} |\hat{V}_0|^2 \left( \frac{R_{\text{rad}}}{R^2 + |X|^2} \right) \]
Linear Dipole in Cold Plasma

Solution to wave equation: \( \vec{E} = j \omega \mu_0 (k \vec{k} - k^2 \vec{I} + \omega^2 \mu_0 \varepsilon) \cdot \vec{J}_{\text{ext}} \).

\[
\left( k \vec{k} - k^2 \vec{I} + \omega^2 \mu_0 \varepsilon \right)^{-1} = \frac{\Lambda}{k^2 \alpha^2 \left( k^2 - k_+^2 \right) \left( k^2 - k_-^2 \right)},
\]

\( \Lambda = k^4 \varepsilon_0 - k^2 k_0^2 L + k_0^4 W, \)

\( \alpha(\theta) = \varepsilon_1 \sin^2 \theta + \varepsilon_3 \cos^2 \theta, \)

\[
\mathbf{L} = \begin{bmatrix}
\varepsilon_1 (n_x^2 + n_y^2) + \varepsilon_3 (n_x^2 + n_z^2) & -j \varepsilon_2 (n_x^2 + n_y^2) + \varepsilon_3 n_x n_y & \varepsilon_1 n_x n_z - j \varepsilon_2 n_y n_z \\
-j \varepsilon_2 (n_x^2 + n_y^2) + \varepsilon_3 n_x n_y & \varepsilon_1 (n_x^2 + n_y^2) + \varepsilon_3 (n_y^2 + n_z^2) & \varepsilon_1 n_y n_z + j \varepsilon_2 n_x n_z \\
\varepsilon_1 n_x n_z + j \varepsilon_2 n_y n_z & \varepsilon_1 n_y n_z - j \varepsilon_2 n_x n_z & \varepsilon_1 (1 + n_z^2)
\end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix}
\varepsilon_1 \varepsilon_3 & -j \varepsilon_2 \varepsilon_3 & 0 \\
-j \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_3 & 0 \\
0 & 0 & \varepsilon_1^2 - \varepsilon_2^2
\end{bmatrix}.
\]

Electric field on external surface of antenna: \( \vec{E}_{\text{ext}} \cdot \hat{\mathbf{j}}_{\text{ext}} \).

Power: \( P = -\frac{1}{2} \int_{V} d^3 x \vec{E} \cdot \vec{J}_{\text{ext}} \).

Surface current on antenna: \( \vec{J}_{\text{ext}} \).

Difficult integral requiring complex k-plane integration to preserve radiation boundary conditions.

Radiation resistance: \( R_{\text{rad}} = \frac{2 \text{Re} \{P\}}{|I_0|^2} \).
Cold Plasma Basics

Maxwell’s equations:
\[ \nabla \cdot \varepsilon_0 \mathbf{E} = \rho, \]
\[ \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right), \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]
\[ \nabla \cdot \mathbf{B} = 0, \]

Equation of motion for species \( i \):
\[ m_i \frac{d\mathbf{v}_i}{dt} = q_i (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \nabla \cdot \mathbf{F}_i, \]

Charge and current density:
\[ \rho = \sum_{i=1}^{N} n_i q_i, \]
\[ \mathbf{J} = \sum_{i=1}^{N} n_i q_i \mathbf{v}_i. \]

Fourier representation:
\[ f(\mathbf{x}, t) = \frac{1}{2} \left[ \tilde{f}(\mathbf{k}, \omega) \exp \{ j(\omega t - \mathbf{k} \cdot \mathbf{x}) \} + c.c. \right], \]

Linear wave equation:
\[ \left( k \mathbf{k} - k^2 \mathbf{I} + \omega^2 \mu_0 \varepsilon_0 \right) \cdot \tilde{\mathbf{E}} = j \omega \tilde{\mathbf{J}}_{\text{ext}}, \]
with dielectric tensor:
\[ \varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_1 & j \varepsilon_2 & 0 \\ -j \varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}, \]
\[ \varepsilon_1 = 1 - \frac{\omega_{pe}^2 U_e}{\omega^2 - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \]
\[ \varepsilon_2 = \frac{-\omega_{pe} \omega_{ce}}{\omega (\omega^2 - \omega_{ce}^2)} + \frac{\omega_{pi} \omega_{ci}}{\omega (\omega^2 - \omega_{ci}^2)}, \]
\[ \varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2}, \]
\[ \omega_{ei} = \frac{|q_i| B_0}{m_{i,e}} \]

Normal modes defined by:
\[ |k \mathbf{k} - k^2 \mathbf{I} + \omega^2 \mu_0 \varepsilon_0 | = A \left( \frac{ck}{\omega} \right)^4 - B \left( \frac{ck}{\omega} \right)^2 + C = 0, \]
with solutions:
\[ A = \varepsilon_1 \sin^2 \theta + \varepsilon_3 \cos^2 \theta, \]
\[ B = (\varepsilon_1^2 - \varepsilon_2^2) \sin^2 \theta + \varepsilon_1 \varepsilon_3 (1 + \cos^2 \theta), \]
\[ C = \varepsilon_3 (\varepsilon_1^2 - \varepsilon_2^2), \]
\[ F^2 = B^2 - 4AC > 0 \]

Lots of different notations:
\[ \varepsilon_1, \varepsilon_2, \varepsilon_3 \leftrightarrow S, D, P, R, L \leftrightarrow X, Y \]
Cold Plasma Mode Structure

Everything determined by:

\[ \omega, \theta, B_0, n_e = n_i \]

CMA Diagram, H+ plasma

Defined by:

\[ 0 = \varepsilon_1 \sin^2 \theta_{res} + \varepsilon_3 \cos^2 \theta_{res} \]

Vacuum limit

“whistler” mode regime

No propagation

Resonance angle is when:

\[ \lambda \rightarrow 0 \ (k \rightarrow \infty) \]
Application to DSX

Assume a fixed linear current profile

**Parallel orientation:**

\[ R_{rad} = \frac{8 \omega \mu_0}{\pi^2 k_0^2 d^2} \int_0^{2\pi} d\phi \int_0^\theta d\theta \frac{\sin \theta}{\alpha \cos^4 \theta} \frac{\Lambda_{33}(k_-)}{k_-^2 (k_-^2 - k_+^2)} \times J_0^2 \left( k_- a \sin \theta \right) \sin^4 \left( \frac{k_- d}{4 \cos \theta} \right). \]

**Perpendicular orientation:**

\[ R_{rad} = \frac{8 \omega \mu_0}{\pi^2 k_0^2 d^2} \int_0^{2\pi} d\phi \int_0^\theta d\theta \frac{1}{\alpha \sin^3 \theta \cos^4 \varphi} \frac{\Lambda_{11}(k_-)}{k_-^2 (k_-^2 - k_+^2)} \times J_0^2 \left( k_- a \left[ 1 - \sin^2 \theta \cos^2 \varphi \right]^{-1/2} \right) \sin^4 \left( \frac{k_- d}{4 \sin \theta \cos \varphi} \right). \]

**DSX parameters:**

- Antenna length \( d \) [m]: 80.0
- Antenna radius \( a \) [m]: 0.1
- Frequency range \( \nu \) [kHz]: 3 - 50
- Free-space wavelength \( \lambda_0 \) [m]: 1.00E+05
- Plasma density \( n_e \) [# cm\(^{-3}\)]: 3.00E+03
- Magnetic field \( B_0 \) [Gauss]: 5.00E-02

(inside the plasmasphere)

Choose cutoff wavelength:

\[ k_{max} = \frac{2\pi}{d / 2} \rightarrow \eta_c^2 = \left( \frac{ck_{max}}{\omega} \right)^2 \rightarrow \tan^2 \theta_c = - \frac{P (\eta_c^2 - R)(\eta_c^2 - L)}{(S\eta_c^2 - RL)(\eta_c^2 - P)} \]

~ sheath size
Antenna Reactance

Electric dipole reactance is due to capacitance

Inside the plasmasphere

In vacuo: \[ C_{ant} = \frac{\pi \varepsilon_0 d}{2 \ln[ d / 2a]} \]

Static sheath (Mlodnosky & Garriott 1962):
\[ C_{sh} = \frac{\pi \varepsilon_0 d}{2 \ln[r_{sh} / a]} \]
\[ 2\varepsilon_0 V_{sh} = -ne^2 r_{sh} \ln[r_{sh} / a] + \frac{ne^2 (r_{sh}^2 - a^2)}{2} \]
\[ V_{sh} = \frac{kT_e}{e} \ln \left[ \frac{v_{sat} + \sqrt{2 / \pi} v_i}{v_e / 2\pi} \right] \]

Dynamic sheath (Song, et al. 2007):
\[ C_{sh} = \frac{\pi \varepsilon_0 d}{\ln[ 2I_\circ / \left( \pi ne_0 d^2 \right) ]} \]

DSX Capacitance

Probably not hugely different than in vacuo
Inside the plasmasphere

“Quality factor” \( Q = \frac{X}{R_{\text{rad}}} \)

Small is good

\[ Q_{\text{vac}}(10 \text{ kHz}) \sim 6.8 \times 10^7 \]

\[ Q_{\text{plasma}}(10 \text{ kHz}) \sim 1.2 \times 10^3 \]

Dipole \( R_{\text{rad}} \) in a plasma look better, at least in the linear limit…
Plasma Antenna Model

**Vacuum** – dipole radiation *in vacuo* out to 10 km then cold plasma propagation

**Linear cold plasma** – voltage and current distribution specified on antenna immersed in a cold plasma

**Self-consistent linear cold plasma** – voltage on terminals specified, current distribution calculated self-consistently for antenna immersed in a cold plasma

Sheath & plasma heating effects – determine sheath capacitance, resistance and antenna-sheath “effective” antenna current

Effective circuit

Total power radiated as function of driving voltage
For an electric dipole: $R_{\text{ant}}$, $Z_L \ll Z_{\text{cap}}$
DSX Pattern Self-Measurement

Lower-Hybrid Settling

wave “mirrors” on constant $L$

Cavity Excitation

Radiated Wavenormals

$\Psi$ @ res. cone

Radiated Wavenormals

$\Psi$ @ res. cone