Inner zone electron radial diffusion coefficients - An update with Van Allen Probes MagEIS data

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Summary

- Estimated inner zone D_{LL} from MagEIS electron flux, L<3.5
  - *Quiet times only (weekly max Kp<4)*
  - *Allowing for pitch angle scattering (loss)*
  - *Rudimentary CRAND (source)*
  - *Neglecting energy transport (D_{EE}): storm times, outside plasmapause*
- Using modified 1st invariant (ζ) and integrating over pitch angle
  - *Obtain 1-D diffusion equation in "bundle content" at fixed ζ*
  - *Solving radial diffusion equation as 1-D ODE for D_{LL}*
- Requires estimate of decay time
  - *Difficult at some L*
  - *Modest effect on D_{LL}*
- Results agree well with Electrostatic D_{LL} from *Schulz* 1991 and *Brautigam and Albert* 2000
  - *Too high for electromagnetic-only DLL, e.g., Ozeke et al., 2014*

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Mathematical Framework

- Modified first invariant $\zeta = \frac{M}{y^2} = \frac{p^2 L^3}{2 m_0 B_0}$
  - *All particles on the field line have same ζ, equal to M for equatorially mirroring particles*
  - *ζ approximately invariant to radial diffusion at fixed M,K*
- Using "Bundle Content"
  - $n(\zeta, L) = \int_0^{x_{LC}} f(\zeta, x, L) x T(y) dx$
  - *Integrates over equatorial pitch angle, weighted by density of states*
  - *Invariant to pitch angle diffusion, except flow into loss cone (boundary flux)*
  - $nL \propto$ *flux tube content*
  - *like omnidirectional flux with different angle weighting*
- The Diffusion Equation is then:
  - $\frac{\partial n}{\partial t} = L^{\frac{5}{2}} \frac{\partial}{\partial L}\Big|_{\zeta} \left[\frac{\bar{D}_{LL}}{L^{\frac{5}{2}}} \frac{\partial n}{\partial L}\Big|_{\zeta} \right] - \frac{n}{\tau} + \bar{S}$
  - *Assumes pitch angle diffusion coefficient and gradient at edge of loss cone depend only on L and ζ and not t*
  - *Angle-averaged radial diffusion $\bar{D}_{LL}$ and CRAND source $\bar{S}$*

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Derivation of D_{LL}

- Start with radial and pitch angle diffusion: $\frac{\partial \bar{f}}{\partial t} = \frac{1}{xT(y)} \frac{\partial}{\partial x}\left[xT(y)D_{xx}\frac{\partial \bar{f}}{\partial x}\right] + L^{\frac{5}{2}}\frac{\partial}{\partial L}\left[\frac{D_{LL}}{L^{\frac{5}{2}}}\frac{\partial \bar{f}}{\partial L}\right] + S$
- Integrate xT(y)dx from equator to loss cone (x_{LC}):
  - $n(\zeta, L) = \int_0^{x_{LC}} \bar{f}(\zeta, x, L) xT(y) dx$
  - $\frac{\partial n}{\partial t} = \left[xT(y)D_{xx}\frac{\partial \bar{f}}{\partial x}\bigg|_{\zeta,L}\right]_{x_{LC}} + L^{\frac{5}{2}}\frac{\partial}{\partial L}\bigg|_{\zeta}\left[\frac{\bar{D}_{LL}}{L^{\frac{5}{2}}}\frac{\partial n}{\partial L}\bigg|_{\zeta}\right] + \bar{S}$
- Assume constant gradient at edge of loss cone and ignore *L* dependence of loss cone angle:
  - $\left[xT(y)D_{xx}\frac{\partial \bar{f}}{\partial x}\bigg|_{\zeta,L}\right]_{x_{LC}} = -\frac{n}{\tau}$
  - $\frac{\partial n}{\partial t} = L^{\frac{5}{2}}\frac{\partial}{\partial L}\bigg|_{\zeta}\left[\frac{\bar{D}_{LL}}{L^{\frac{5}{2}}}\frac{\partial n}{\partial L}\bigg|_{\zeta}\right] - \frac{n}{\tau} + \bar{S}$
- Solve for D_{LL}: $\bar{D}_{LL} = D_0 L_1^m + \left(\frac{\partial n}{\partial L}\bigg|_{\zeta}\right)^{-1} L^{\frac{5}{2}} \int_{L_1}^{L}\left[\frac{\partial n}{\partial t} + \frac{n}{\tau} - \bar{S}\right] L^{-\frac{5}{2}} dL$
  - *At low L, fit* $\bar{D}_{LL} = D_0 L_1^m$ *to obtain* D_0,*m*

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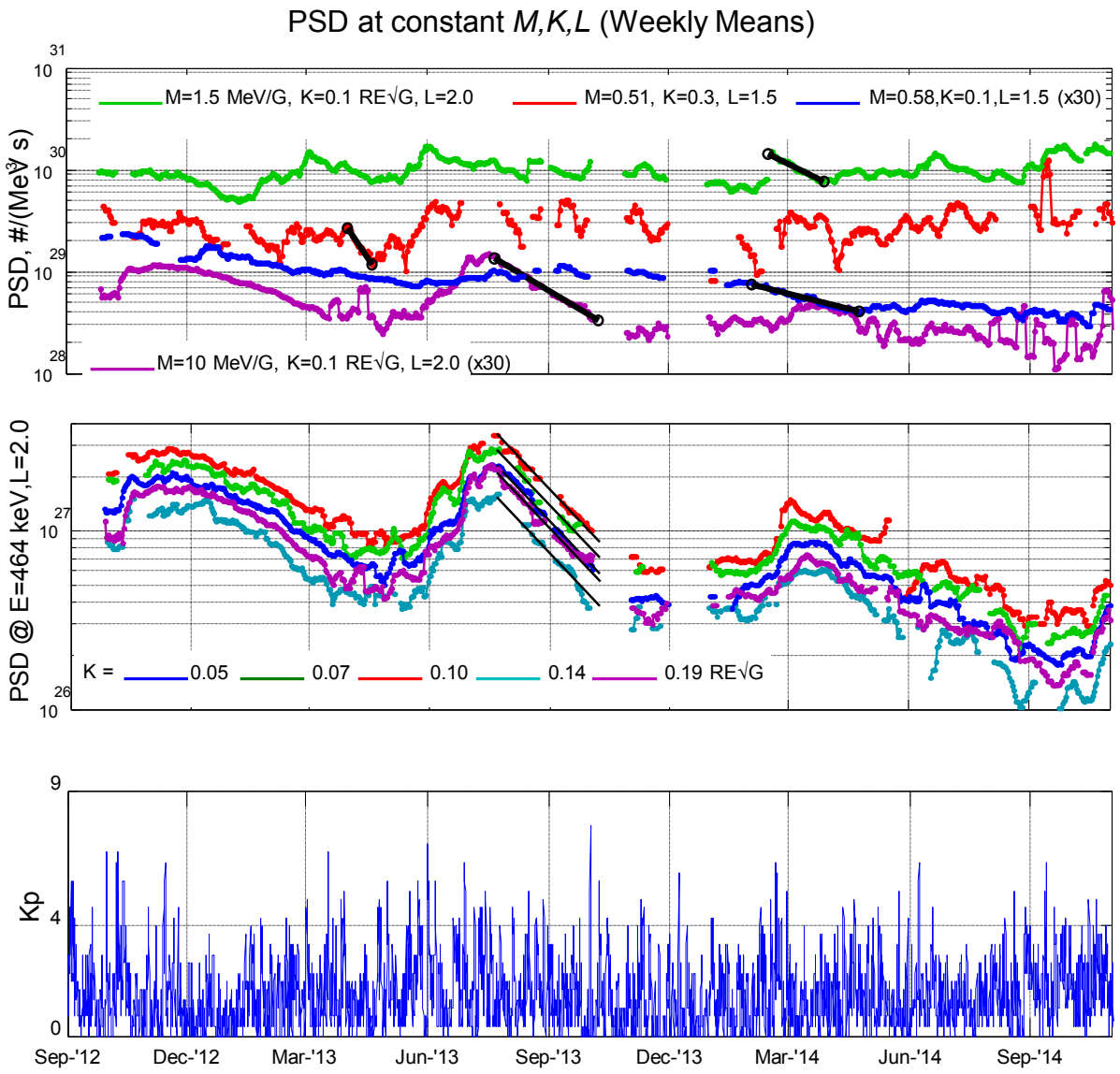
Time Series of PSD

- Periods of roughly exponential decay interrupted by sudden or gradual rises
- Pitch angle distribution is not changing a lot
- Especially stable during roughly exponential decays



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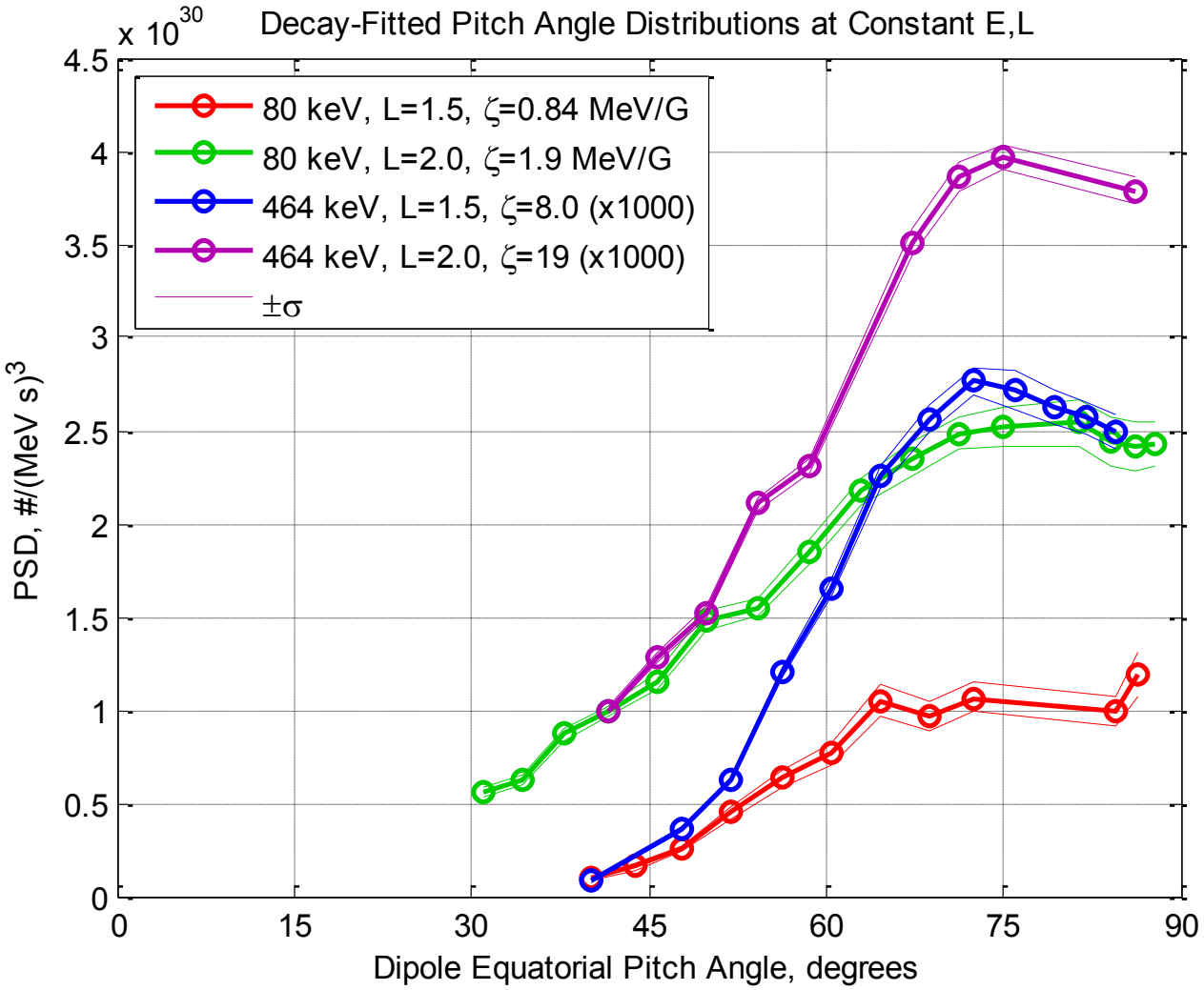
Pitch angle distributions

- These are the pitch angle distributions at fixed energy during decays at L=1.5 and L=2
- At 464 keV, evidence of off-90 peak – exponentially decaying, but not(!) an eigenmode of angular scattering
- At 80 keV, evidence of off-90 peak is less clear



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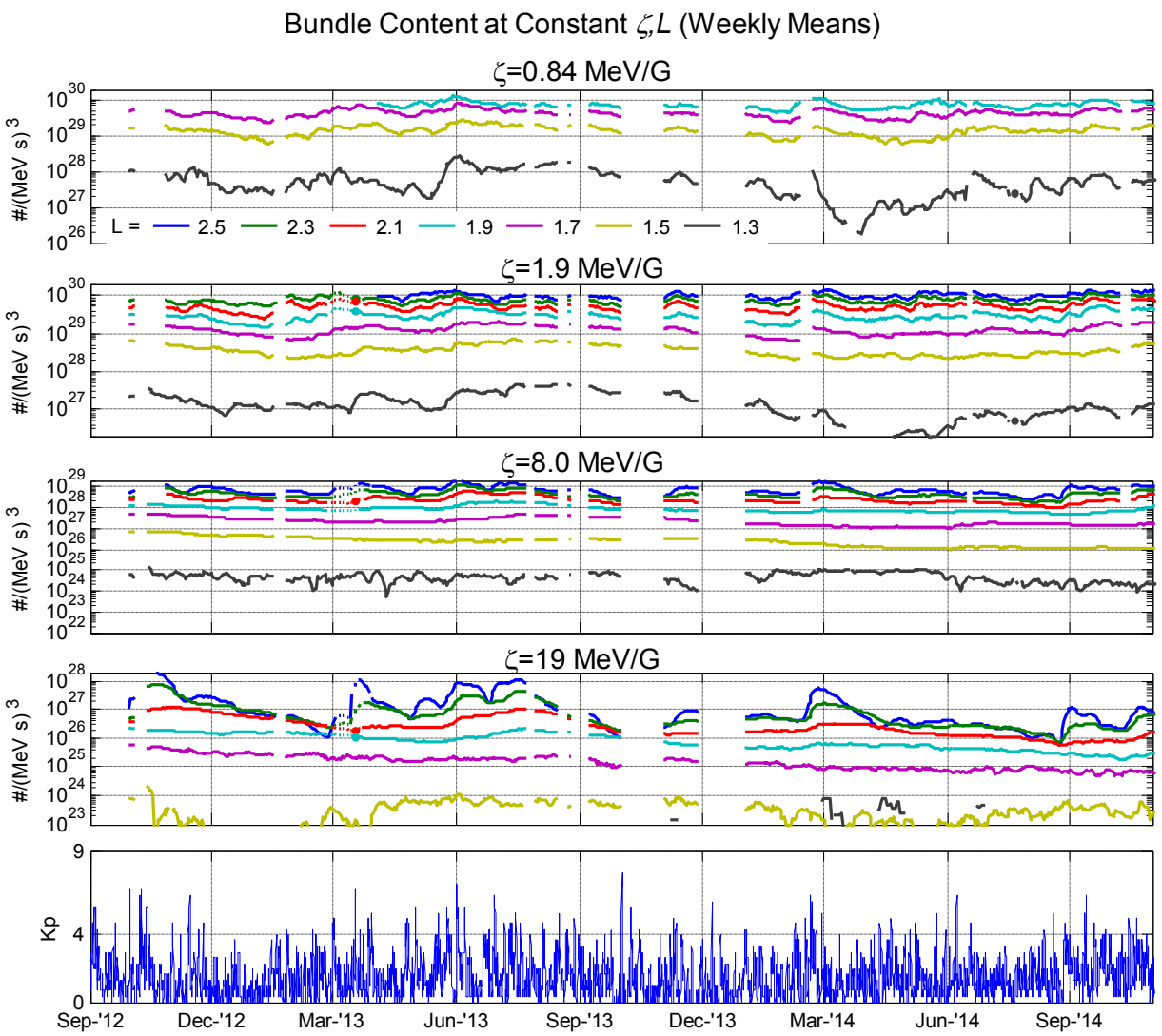
Time Series of Bundle Content

- Fast variations
  - *Only affects highest Ls*
  - *More dramatic at higher energy (ζ)*
  - *Caused by storms*
- Slow variations
  - *Pitch angle scattering into atmosphere (~exponential decay)*
  - *Radial transport (undulations)*
- Compute *dn/dL* and *dn/dt* on weekly timescales



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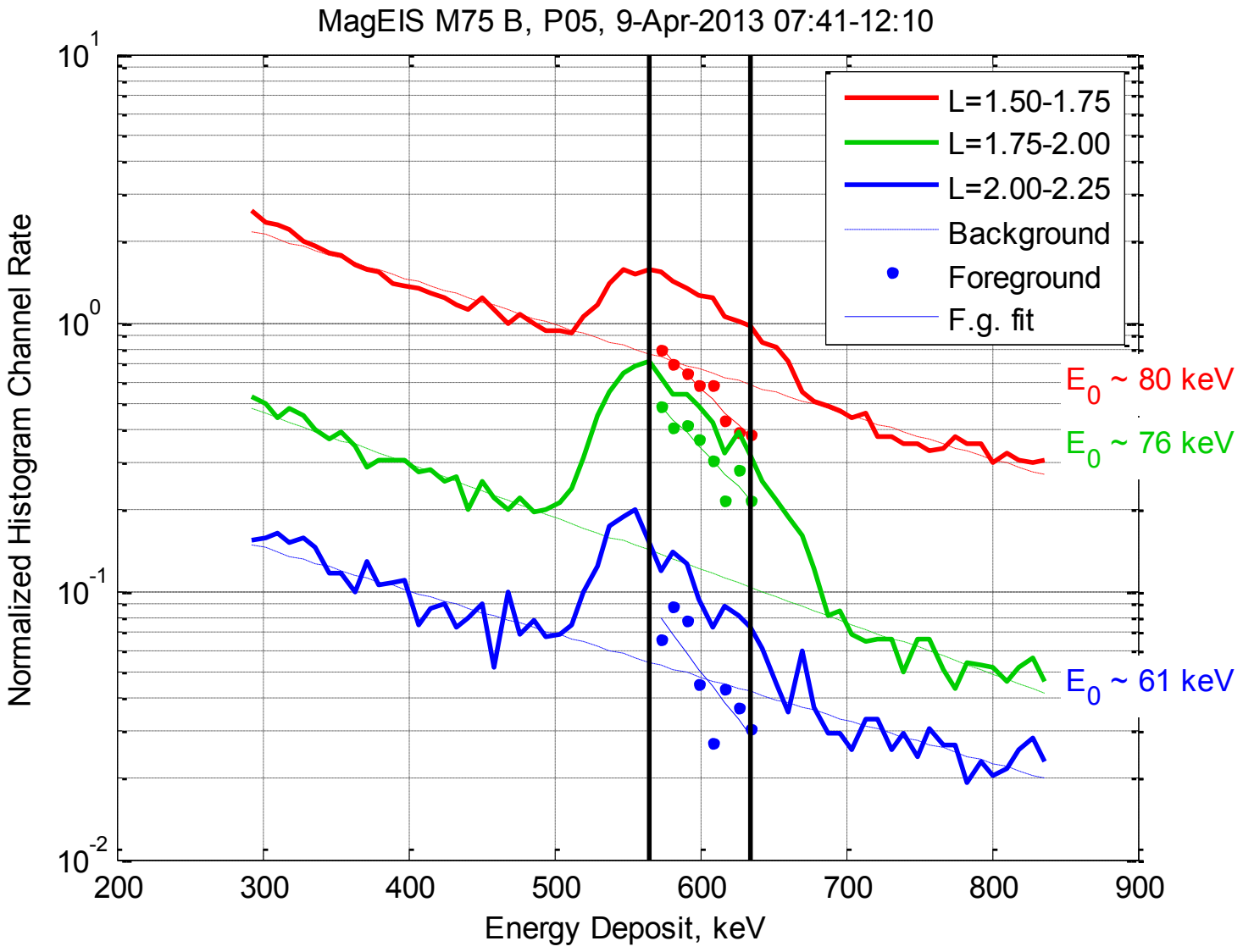
Energy Diffusion Can Be Neglected at Low L



- MagEIS histogram data
- We background correct the L-binned histogram data
- Then we fit to an exponential assuming the energy deposit is the incident energy
- Here we see energy gradients with exponential fall-off with a constant of 60-80 keV
- **The steep energy gradient would not persist if energy diffusion were significant**

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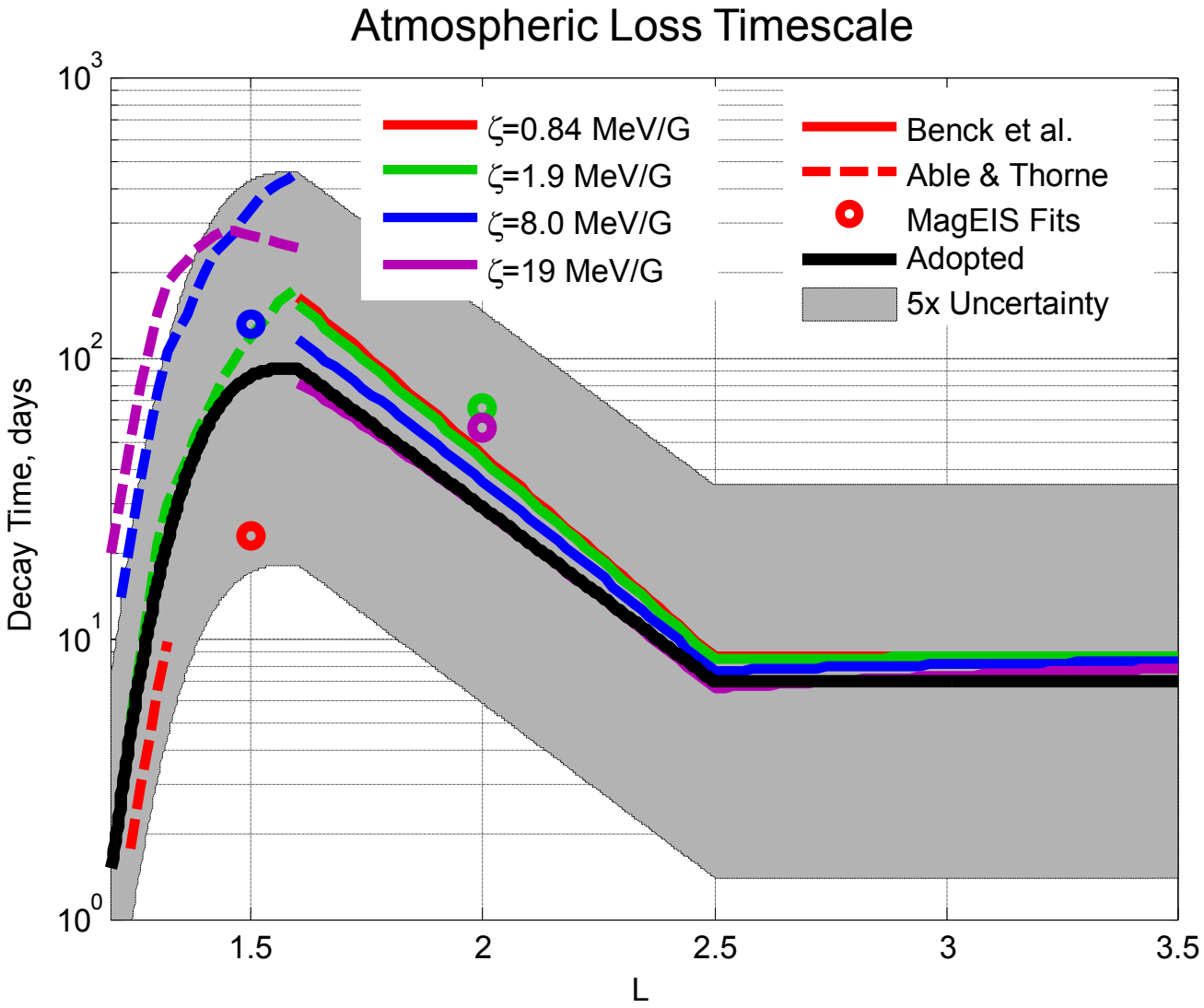
Decay Time is Uncertain



- There are several estimates of decay time, including my own fits to the periods in the previous slides
- None of these estimates agree well with each other
- I have adopted a decay time that matches Benke et al. [2010] at ζ=11 for L=1.5 but follows the Abel and Thorne [1998] shape at ζ=11 below that.
- Assumed decay time does not vary with ζ
- To account for uncertainty in τ, evaluate D_{LL} for the nominal τ and for 5τ and $\tau/5$.

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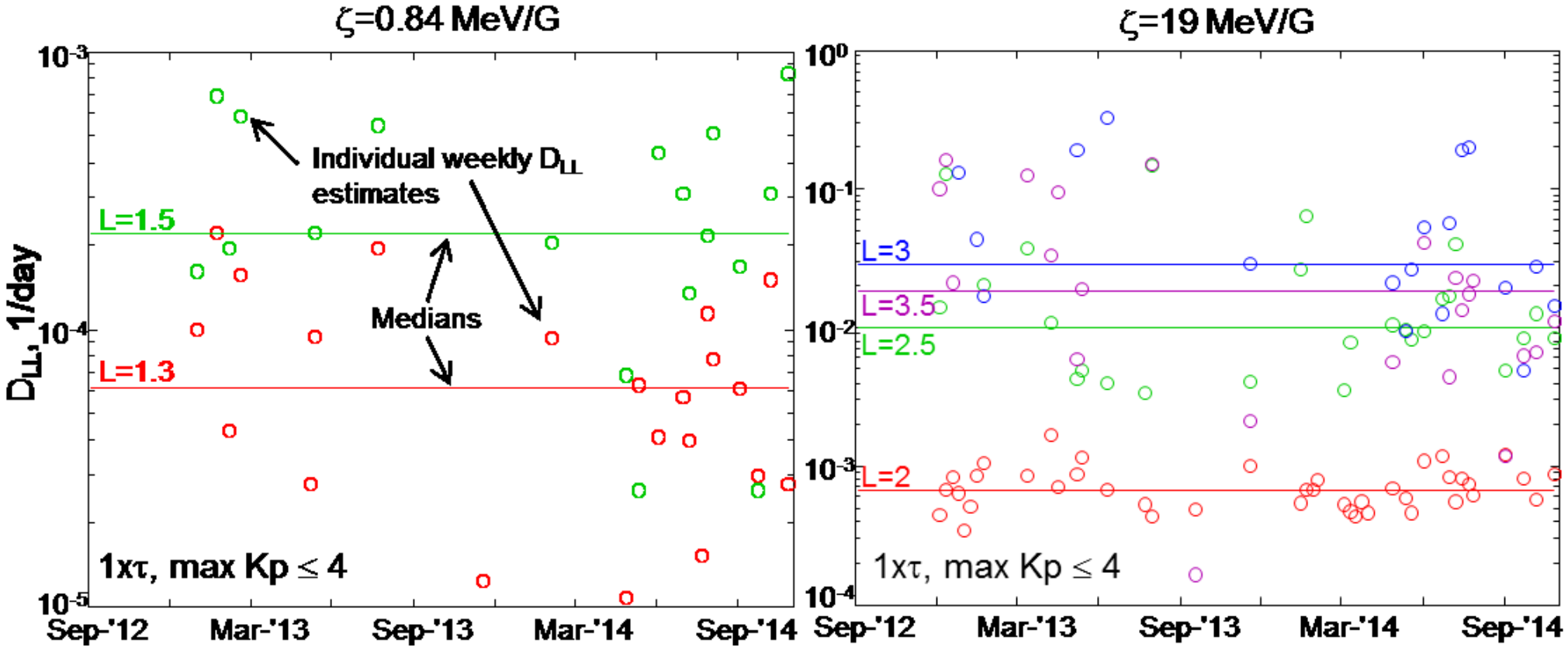
D_{LL} versus time



- Max(Kp)>4, D_{LL}<0 and *dn/dt*<n/100 filtered out
- Variation in D_{LL} associated with (noisy) *dn/dt*
- Use Median as fiducial D_{LL}
- Keep percentiles as error bars
- (Note these plots are for nominal τ, we also do 5τ and τ/5)

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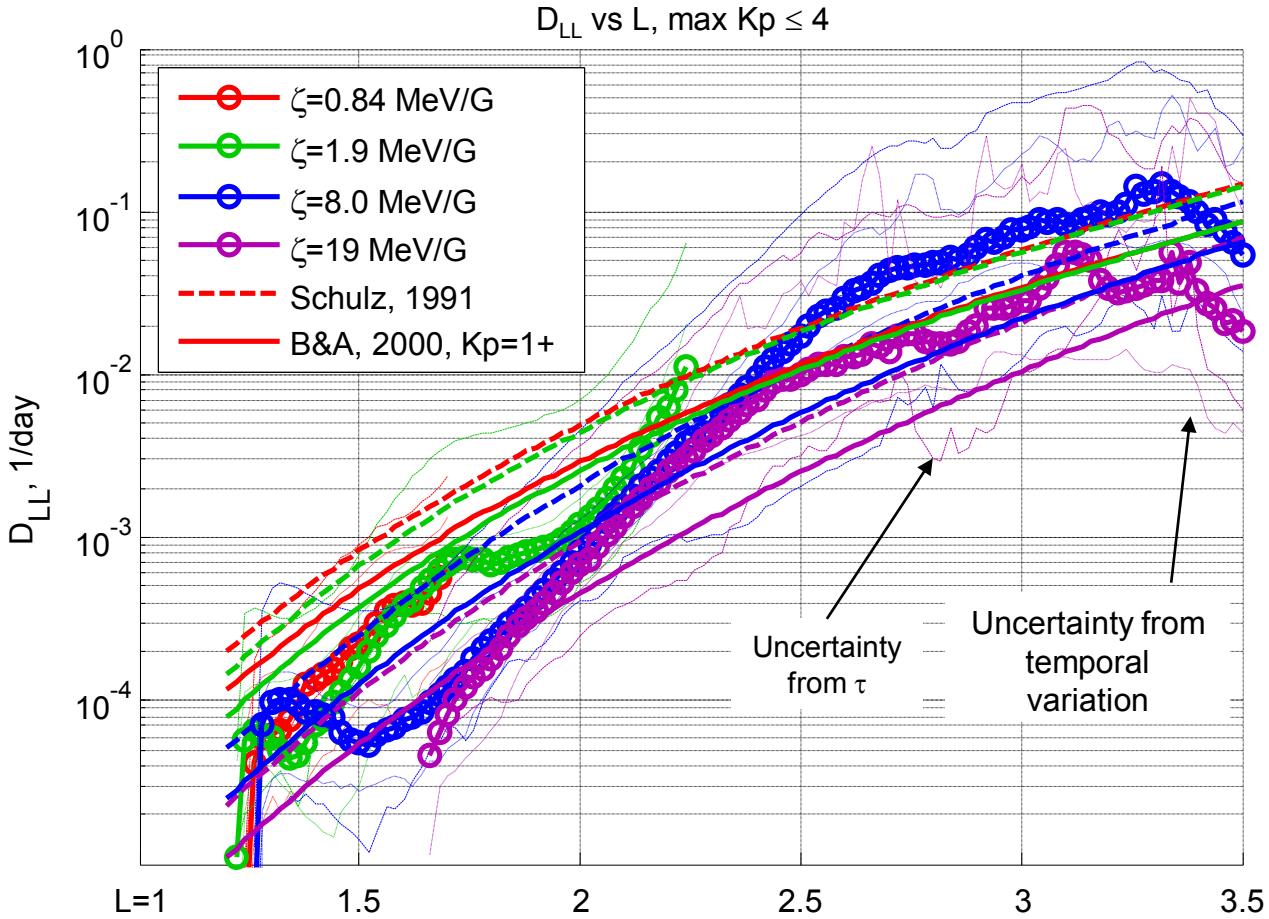
D_{LL} vs L

- No D_{LL} for ζ=25, L<1.7 due to low counts/background
- D_{LL} for ζ=1,2 ends at L=2.4, 3 due to energy dropping below 50 keV
- D_{LL} matches *Schulz* [1991] and *Brautigam and Albert* [2000] estimates
  - *Dominated by electrostatic (L^6)*
  - *Electromagnetic (L^{10}) negligible at low L*



(ζ = 19) 1970 keV	906	488	288	181	120
(ζ = 8.0) 1159 keV	494	249	139	85	55
(ζ = 1.9) 426 keV	155	71	37	22	14
(ζ = 0.84) 214 keV	72	31	16	10	6

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A note on the electrostatic D_{LL} model

- Assume a series of exponentially decaying impulses in convection electric field:
  - $E_c(t') = E_c(t) + \Delta E_c e^{-(t'-t)/\tau_d}$
  - *Substorms and/or changes in IMF driving*
  - – *With a squared amplitude per unit time of:* $\frac{1}{\tau}\sum(\Delta E_c)^2$
- This gives:
  - $D_{LL} = 2\left(\frac{c}{4aB_0}\right)^2 L^6 \mathcal{E}_c(\Omega_3/2\pi),\ \mathcal{E}_c(\omega/2\pi) = \frac{2(\tau_d^2/\tau)\sum(\Delta E_c)^2}{1+\omega^2\tau_d^2}$
  - $D_{LL} = \left(\frac{2\pi qa}{3B_0}\right)^2 [(1/\tau)\sum(\Delta E_c)^2] L^{10}\left(\frac{\gamma y^2}{M}\right)^2\left(\frac{T(y)}{2D(y)}\right)^2 \frac{1}{1+(\tau_d\Omega_3)^{-2}}$
  - $D_{LL} = (10^{-10}\ \mathrm{day}^{-1}) L^{10}\left(\frac{\gamma M_0 y^2}{M}\right)^2\left(\frac{T(y)}{2D(y)}\right)^2 \frac{1}{1+(\tau_d\Omega_3)^{-2}}$ *(M_0=1 GeV/G, τ_d=1200 s)*
  - *Normalization derived from particle dynamics, e.g., Croley et al. [1976], who studied 10s MeV protons on OV1-19*
  - *Similar answers obtained from estimates of* $[(1/\tau)\sum(\Delta E_c)^2]$*, e.g., Mozer [1971]*
- The implied electric field variation is:
  - $[(1/\tau)\sum(\Delta E_c)^2] = 2.5\times 10^{-7}$ (mV/m)^2/s
  - ΔE_c*~0.1 mV/m every ~11 hours or* ΔE_c*~0.01 mV/m every ~7 minutes*
  - *4 orders of magnitude smaller than storm time model from* Chen and Schulz *[1992], neglecting shielding; consistent with early quiet time estimates*

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Conclusion & Discussion

- Inner zone D_{LL} estimates are consistent with prior fits
  - *Schulz [1991] (tied back to in situ 10s MeV proton dynamics and ~MeV electron dynamics)*
  - *Brautigam and Albert [2000] (tied back to observed changes in cross-polar cap potential)*

- Some simulations use only electromagnetic D_{LL}, e.g., *Ozeke et al.*, [2014]
  - *Dominant in the outer zone*
  - *Negligible in inner zone*

- Simulators should include electrostatic D_{LL} for inner zone studies

- We can extrapolate inner zone D_{LL} to outer zone to subtract electrostatic part from D_{LL}^{Etotal}

- Concepts like “Bundle Content” allow us to reduce dimension of the system: specify boundary fluxes rather than PSD over extra dimension

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