Inner zone electron radial diffusion coefficients - An update with Van Allen Probes MagEIS data


The Aerospace Corporation
El Segundo, CA

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Summary

• Estimated inner zone $D_{LL}$ from MagEIS electron flux, L<3.5
  – Quiet times only (weekly max Kp<4)
  – Allowing for pitch angle scattering (loss)
  – Rudimentary CRAND (source)
  – Neglecting energy transport ($D_{EE}$): storm times, outside plasmapause

• Using modified 1st invariant ($\zeta$) and integrating over pitch angle
  – Obtain 1-D diffusion equation in “bundle content” at fixed $\zeta$
  – Solving radial diffusion equation as 1-D ODE for $D_{LL}$

• Requires estimate of decay time
  – Difficult at some L
  – Modest effect on $D_{LL}$

• Results agree well with Electrostatic $D_{LL}$ from Schulz 1991 and Brautigam and Albert 2000
  – Too high for electromagnetic-only DLL, e.g., Ozeke et al., 2014
Mathematical Framework

• Modified first invariant $\zeta = \frac{M}{y^2} = \frac{p^2L^3}{2m_0B_0}$
  - All particles on the field line have same $\zeta$, equal to $M$ for equatorially mirroring particles
  - $\zeta$ approximately invariant to radial diffusion at fixed $M, K$

• Using “Bundle Content”
  - $n(\zeta, L) = \int_0^{xLc} f(\zeta, x, L)xT(y)dx$
  - Integrates over equatorial pitch angle, weighted by density of states
  - Invariant to pitch angle diffusion, except flow into loss cone (boundary flux)
  - $nL \propto$ flux tube content
  - like omnidirectional flux with different angle weighting

• The Diffusion Equation is then:
  $$\frac{\partial n}{\partial t} = L^2 \frac{\partial}{\partial L} \left[ \frac{\bar{D}_{LL}}{L^2} \frac{\partial n}{\partial \zeta} \right] - \frac{n}{\tau} + \bar{S}$$
  - Assumes pitch angle diffusion coefficient and gradient at edge of loss cone depend only on $L$ and $\zeta$
  - and not $t$
  - Angle-averaged radial diffusion $\bar{D}_{LL}$ and CRAND source $\bar{S}$
Derivation of $D_{LL}$

- Start with radial and pitch angle diffusion:
  \[
  \frac{\partial \tilde{f}}{\partial t} = \frac{1}{xT(y)} \frac{\partial}{\partial x} \left[ xT(y)D_{xx} \frac{\partial \tilde{f}}{\partial x} \right] + L^2 \frac{\partial}{\partial L} \left[ \frac{D_{LL}}{L^2} \frac{\partial \tilde{f}}{\partial L} \right] + S
  \]

- Integrate $xT(y)dx$ from equator to loss cone ($x_{LC}$):
  \[
  n(\zeta, L) = \int_0^{x_{LC}} \tilde{f}(\zeta, x, L)xT(y)dx
  \]
  \[
  \frac{\partial n}{\partial t} = \left[ xT(y)D_{xx} \frac{\partial \tilde{f}}{\partial x} \right]_{x_{LC}} + L^2 \frac{\partial}{\partial \zeta} \left[ \frac{D_{LL}}{L^2} \frac{\partial n}{\partial \zeta} \right] + \tilde{S}
  \]

- Assume constant gradient at edge of loss cone and ignore $L$ dependence of loss cone angle:
  \[
  \left[ xT(y)D_{xx} \frac{\partial \tilde{f}}{\partial x} \right]_{x_{LC}} = -\frac{n}{\tau}
  \]

  \[
  \frac{\partial n}{\partial t} = L^2 \frac{\partial}{\partial \zeta} \left[ \frac{D_{LL}}{L^2} \frac{\partial n}{\partial \zeta} \right] - \frac{n}{\tau} + \tilde{S}
  \]

- Solve for $D_{LL}$: $\bar{D}_{LL} = D_0L_1^m + \left( \frac{\partial n}{\partial L} \right)^{-1} L^2 \int_0^L \left[ \frac{\partial n}{\partial t} + \frac{n}{\tau} - \tilde{S} \right] L^{-2} dL$

  - At low $L$, fit $\bar{D}_{LL} = D_0L_1^m$ to obtain $D_0, m$
Time Series of PSD

- Periods of roughly exponential decay interrupted by sudden or gradual rises
- Pitch angle distribution is not changing a lot
- Especially stable during roughly exponential decays
Pitch angle distributions

- These are the pitch angle distributions at fixed energy during decays at L=1.5 and L=2
- At 464 keV, evidence of off-90 peak – exponentially decaying, but not(!) an eigenmode of angular scattering
- At 80 keV, evidence of off-90 peak is less clear
Time Series of Bundle Content

- Fast variations
  - Only affects highest Ls
  - More dramatic at higher energy ($\zeta$)
  - Caused by storms
- Slow variations
  - Pitch angle scattering into atmosphere (~exponential decay)
  - Radial transport (undulations)
- Compute $dn/dL$ and $dn/dt$ on weekly timescales
Energy Diffusion Can Be Neglected at Low L

- MagEIS histogram data
- We background correct the L-binned histogram data
- Then we fit to an exponential assuming the energy deposit is the incident energy
- Here we see energy gradients with exponential fall-off with a constant of 60-80 keV
- The steep energy gradient would not persist if energy diffusion were significant
Decay Time is Uncertain

- There are several estimates of decay time, including my own fits to the periods in the previous slides.
- None of these estimates agree well with each other.
- I have adopted a decay time that matches Benke et al. [2010] at $\zeta=11$ for $L=1.5$ but follows the Abel and Thorne [1998] shape at $\zeta=11$ below that.
- Assumed decay time does not vary with $\zeta$.
- To account for uncertainty in $\tau$, evaluate $D_{LL}$ for the nominal $\tau$ and for $5\tau$ and $\tau/5$. 

**Atmospheric Loss Timescale**

- $\zeta=0.84$ MeV/G
- $\zeta=1.9$ MeV/G
- $\zeta=8.0$ MeV/G
- $\zeta=19$ MeV/G

- Benck et al.
- Able & Thorne
- MagEIS Fits
- Adopted
- 5x Uncertainty
• Max(Kp)>4, $D_{LL}<0$ and $dn/dt<n/100$ filtered out

• Variation in $D_{LL}$ associated with (noisy) $dn/dt$

• Use Median as fiducial $D_{LL}$

• Keep percentiles as error bars

• (Note these plots are for nominal $\tau$, we also do $5\tau$ and $\tau/5$)
\(D_{LL} \text{ vs } L\)

- No \(D_{LL}\) for \(\zeta = 25, L < 1.7\) due to low counts/background
- \(D_{LL}\) for \(\zeta = 1, 2\) ends at \(L = 2.4, 3\) due to energy dropping below 50 keV
- \(D_{LL}\) matches Schulz [1991] and Brautigam and Albert [2000] estimates
  - Dominated by electrostatic \((L^6)\)
  - Electromagnetic \((L^{10})\) negligible at low \(L\)
A note on the electrostatic $D_{LL}$ model

- Assume a series of exponentially decaying impulses in convection electric field:
  \[ E_c(t') = E_c(t) + \Delta E_c e^{-(t'-t)/\tau_d} \]
- Substorms and/or changes in IMF driving
  - With a squared amplitude per unit time of \( \frac{1}{\tau} \sum (\Delta E_c)^2 \)

This gives:

- \[ D_{LL} = 2 \left( \frac{c}{4 a B_0} \right)^2 L^6 \epsilon_c \left( \Omega_3 / 2\pi \right), \quad \epsilon_c (\omega / 2\pi) = \frac{2 (\tau_d^2 / \tau) \sum (\Delta E_c)^2}{1 + \omega^2 \tau_d^2} \]
- \[ D_{LL} = \left( \frac{2 \pi q a^2}{3 B_0} \right)^2 \left[ (1/\tau) \sum (\Delta E_c)^2 \right] L^{10} \left( \frac{\gamma y^2}{M} \right)^2 \left( \frac{T(\gamma)}{2D(y)} \right)^2 \frac{1}{1 + (\tau_d \Omega_3)^2} \]
- \[ D_{LL} = (10^{-10} \text{ day}^{-1}) L^{10} \left( \frac{\gamma M_0 y^2}{M} \right)^2 \left( \frac{T(\gamma)}{2D(y)} \right)^2 \frac{1}{1 + (\tau_d \Omega_3)^2} \quad (M_0 = 1 \text{ GeV/G}, \tau_d = 1200 \text{ s}) \]
- Normalization derived from particle dynamics, e.g., Croley et al. [1976], who studied 10s MeV protons on OV1-19
  - Similar answers obtained from estimates of \( (1/\tau) \sum (\Delta E_c)^2 \), e.g., Mozer [1971]

The implied electric field variation is:

- \( (1/\tau) \sum (\Delta E_c)^2 \approx 2.5 \times 10^{-7} \text{ (mV/m)}^2 / \text{s} \)
- \( \Delta E_c \approx 0.1 \text{ mV/m every } \approx 11 \text{ hours or } \Delta E_c \approx 0.01 \text{ mV/m every } \approx 7 \text{ minutes} \)
- 4 orders of magnitude smaller than storm time model from Chen and Schulz [1992], neglecting shielding; consistent with early quiet time estimates
Conclusion & Discussion

- Inner zone $D_{LL}$ estimates are consistent with prior fits
  - Schulz [1991] (tied back to in situ 10s MeV proton dynamics and ~MeV electron dynamics)
  - Brautigam and Albert [2000] (tied back to observed changes in cross-polar cap potential)

- Some simulations use only electromagnetic $D_{LL}$, e.g., Ozeke et al., [2014]
  - Dominant in the outer zone
  - Negligible in inner zone

- Simulators should include electrostatic $D_{LL}$ for inner zone studies

- We can extrapolate inner zone $D_{LL}$ to outer zone to subtract electrostatic part from $D_{LL}^{E_{total}}$

- Concepts like “Bundle Content” allow us to reduce dimension of the system: specify boundary fluxes rather than PSD over extra dimension