Anomalous radial diffusion by pitch-angle scattering on split drift shells: Calculations

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RBSP Science Working Group Meeting, 23-24 May 2011

This research was funded by NASA HGIP Grant NNX10AQ51G

Outline

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- Hypothesis: pitch-angle diffusion leads to radial diffusion
- Assumed waves
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- Calculated magnetic drift shell splitting
- Estimated magnitude of anomalous radial diffusion
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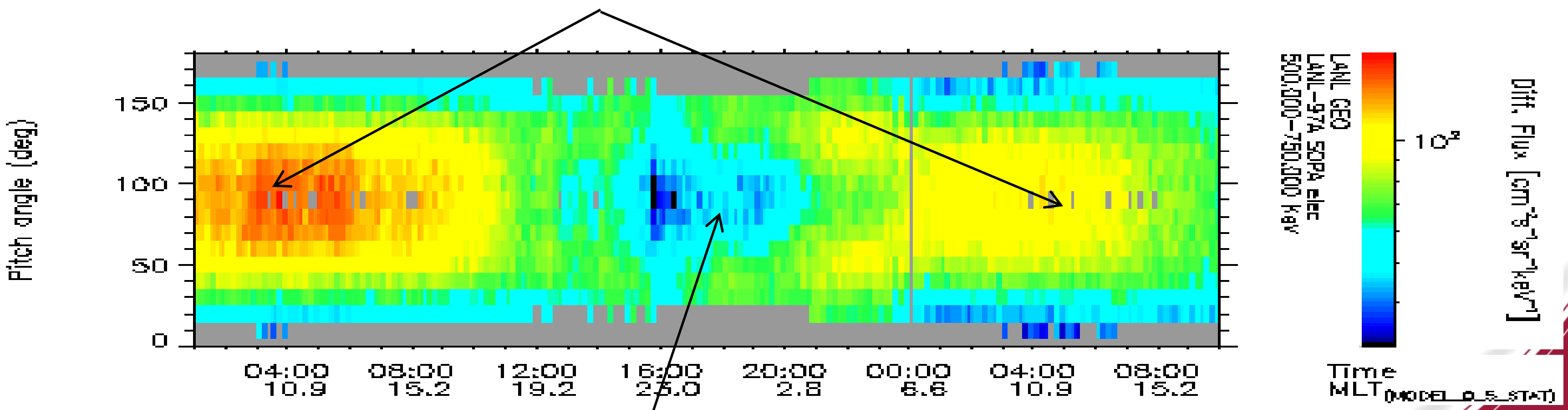
Motivation

1. Global electron radiation belt simulations often rely on diffusion in adiabatic coordinates in a 2.5-D approach
   - *2-D diffusion in E, α_{eq} (M,K),*
   - *Separable 1-D diffusion in L*
   - *No L-E or L-α_{eq} cross terms (L-M or L-K)*
2. Drift-shell splitting observed at/near GEO
3. Pitch-angle scattering is thought to be fast compared to drift-resonant radial diffusion

“trapped” shape at noon

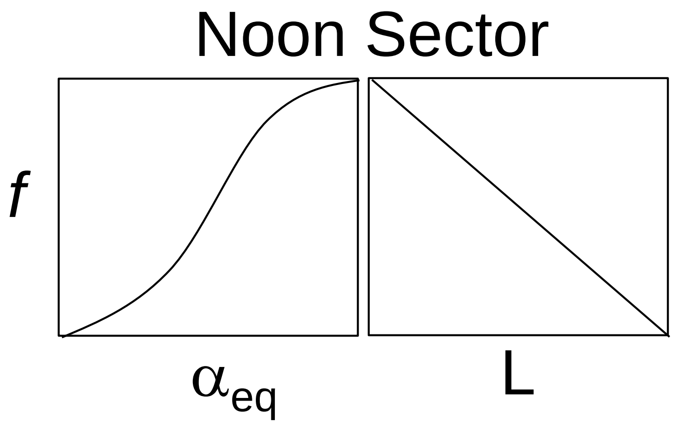


“butterfly” at midnight

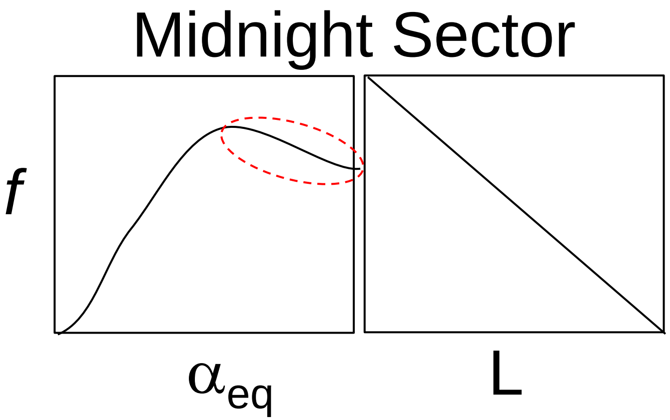
Gradients in L versus gradients in α_{eq}

$$\frac{df}{d\alpha_0} = \left(\frac{\partial f}{\partial K}\right)_{M,L} \frac{dK}{d\alpha_{eq}} + \left(\frac{\partial f}{\partial L}\right)_{M,K} \frac{dL}{d\alpha_{eq}}$$

<0 <0 <0 ?



$$\frac{dL}{d\alpha_{eq}} < 0$$



$$\frac{dL}{d\alpha_{eq}} > 0$$

- Funny pitch angle distributions may reverse the local direction of diffusion in pitch angle. However the direction of diffusion in K&L is the same at all locations on the drift orbit, *unless there is drift phase structure in the f(M,K,L)*
- Thus, drift-averaged diffusion remains a valid description, if one uses drift invariants (D_{MM}, D_{KK}, D_{LL}, etc.) and there is no drift phase structure

Hypothesis: pitch angle diffusion leads to radial diffusion

$$\hat{D}_{LL}^{S}\left(\vec{r}\right)=\hat{D}_{\alpha\alpha}\left(\vec{r}\right)\left(\left.\frac{\partial L}{\partial \alpha}\right|_{\vec{r}}\right)^{2}$$

- Pitch angle diffusion via cyclotron resonance is typically fast compared to radial diffusion
- Magnetic drift shell splitting leads to a dependence of *L* (more precisely *L**) on pitch angle
- Together, these two effects lead to violation of the third invariant (*L*) and radial diffusion
- This process breaks *all* the invariants: there are also anomalous cross-diffusion terms $\hat{D}_{KL}^{S}\left(\vec{r}\right)=\hat{D}_{\alpha\alpha}\left(\vec{r}\right)\left.\frac{\partial L}{\partial \alpha}\right|_{\vec{r}}\left.\frac{\partial K}{\partial \alpha}\right|_{\vec{r}}$ $\hat{D}_{ML}^{S}\left(\vec{r}\right)=\hat{D}_{\alpha\alpha}\left(\vec{r}\right)\left.\frac{\partial L}{\partial \alpha}\right|_{\vec{r}}\left.\frac{\partial M}{\partial \alpha}\right|_{\vec{r}}+\hat{D}_{\alpha p}\left(\vec{r}\right)\left.\frac{\partial L}{\partial \alpha}\right|_{\vec{r}}\left.\frac{\partial M}{\partial p}\right|_{\vec{r}}$
- A formal treatment can be found in *Schulz and Lanzerotti [1974] Section III.7*

Approach

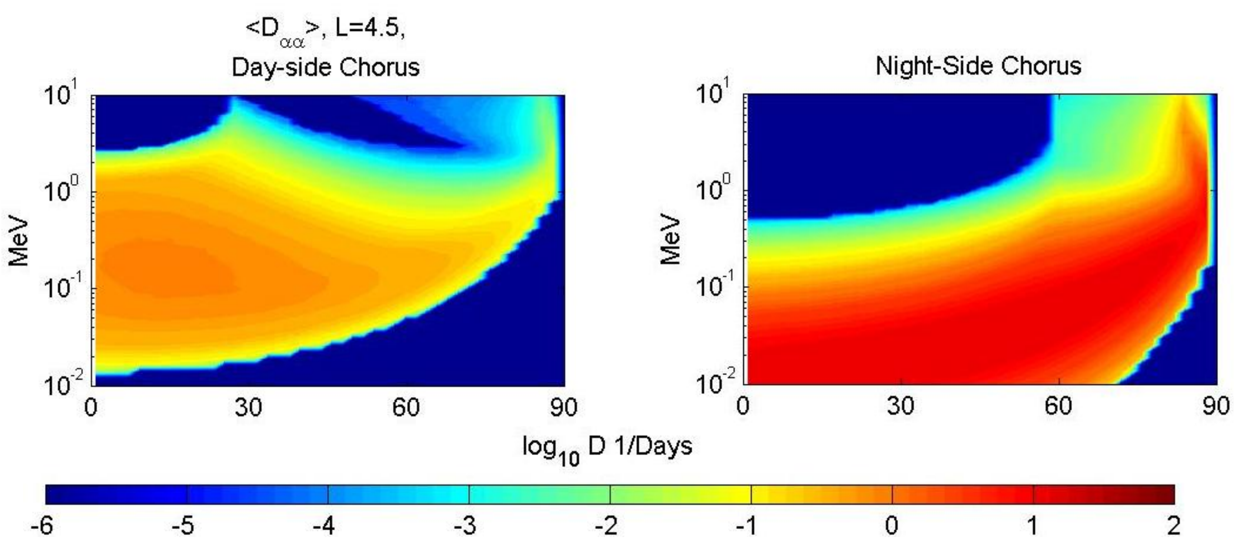
- We compute the drift-bounce-averaged anomalous DLL as:

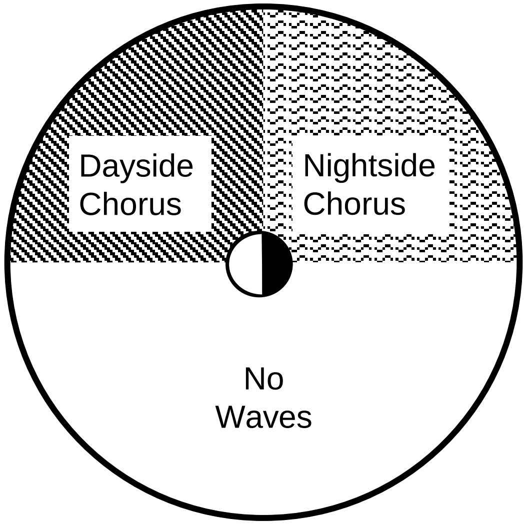
$$D_{LL}^{S} = \left\langle D_{\alpha\alpha}\left(\frac{\partial L}{\partial \alpha}\right)^2 \right\rangle \approx \left\langle D_{\alpha eq \alpha eq} \right\rangle \left\langle \left(\frac{\partial L}{\partial \alpha_{eq}}\right)^2 \right\rangle$$

- We used $D\alpha_{eq}\alpha_{eq}$ from Shprits et al., 2007 at L=4.5
  - Computed with the field aligned approximation in a dipole
  - Omitted EMIC and hiss in plumes because they are concentrated near Dusk, where $dL^*/d\alpha_{eq}$ is small
  - Because $D\alpha_{eq}\alpha_{eq}$ was computed in a dipole field, it is drift invariant within the MLT of the waves, and so the equation above is exact with proper MLT weighting
- We drift-averaged $dL^*/d\alpha_{eq}$ computed in T89 with Kp=4 for $\alpha_{eq} \sim 65^o$ in the vicinity of GEO

Pitch-Angle Diffusion Coefficients

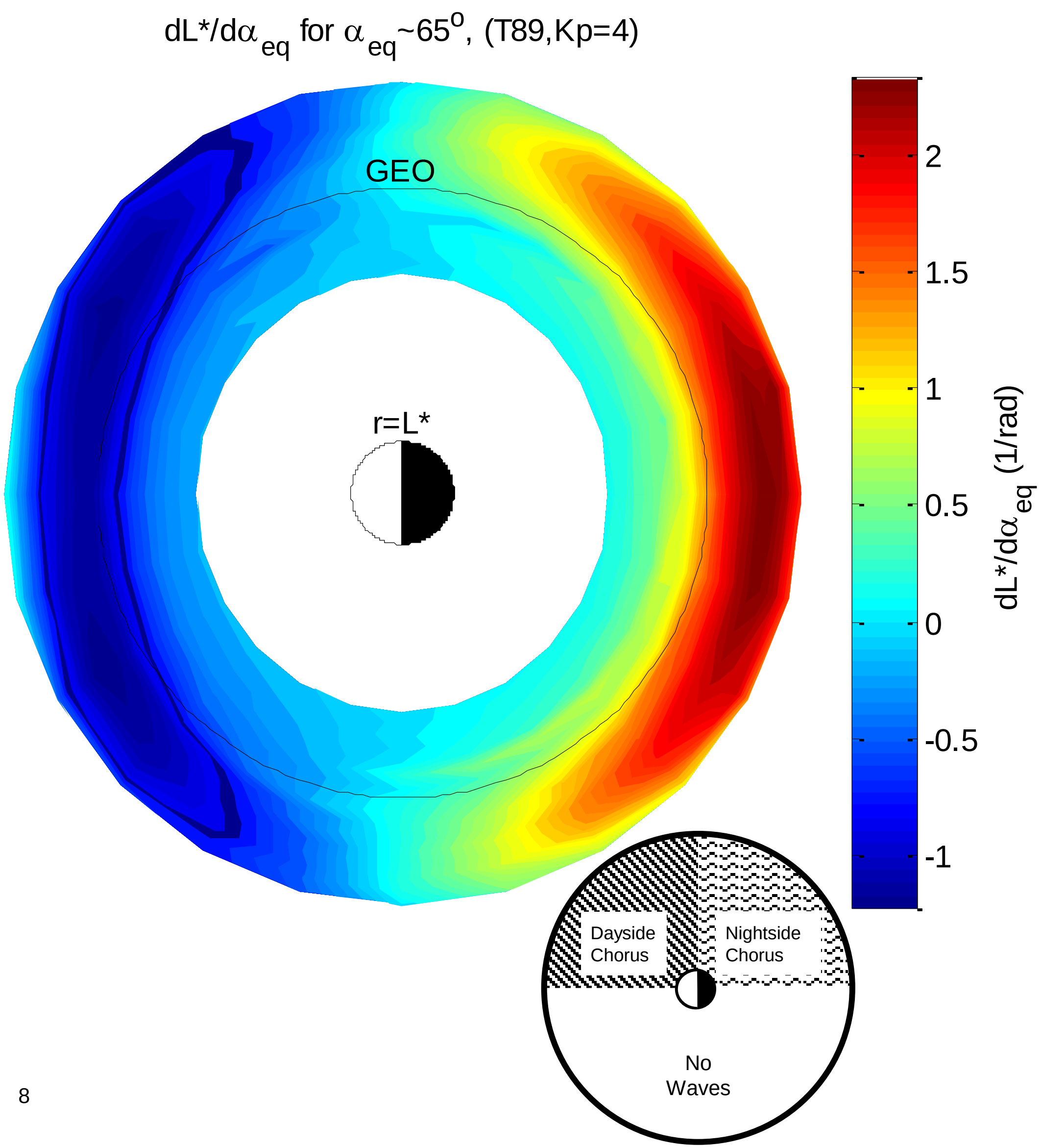
Type of wave	B_W	λ_{max}	Density model	MLT	Wave spectral properties
Chorus day	$10^{0.75+0.04\lambda}$pT	35°	*Sheeley* et al. [2001]	06-12	$\omega_m/\Omega_e = 0.2$, $\delta\omega/\Omega_e = 0.1$, $\omega_{uc}/\Omega_e = 0.3$, $\omega_{lc}/\Omega_e = 0.1$
Chorus night	50 pT	15°	*Sheeley* et al. [2001]	00-06	$\omega_m/\Omega_e = 0.35$, $\delta\omega/\Omega_e = 0.15$, $\omega_{uc}/\Omega_e = 0.65$, $\omega_{lc}/\Omega_e = 0.05$





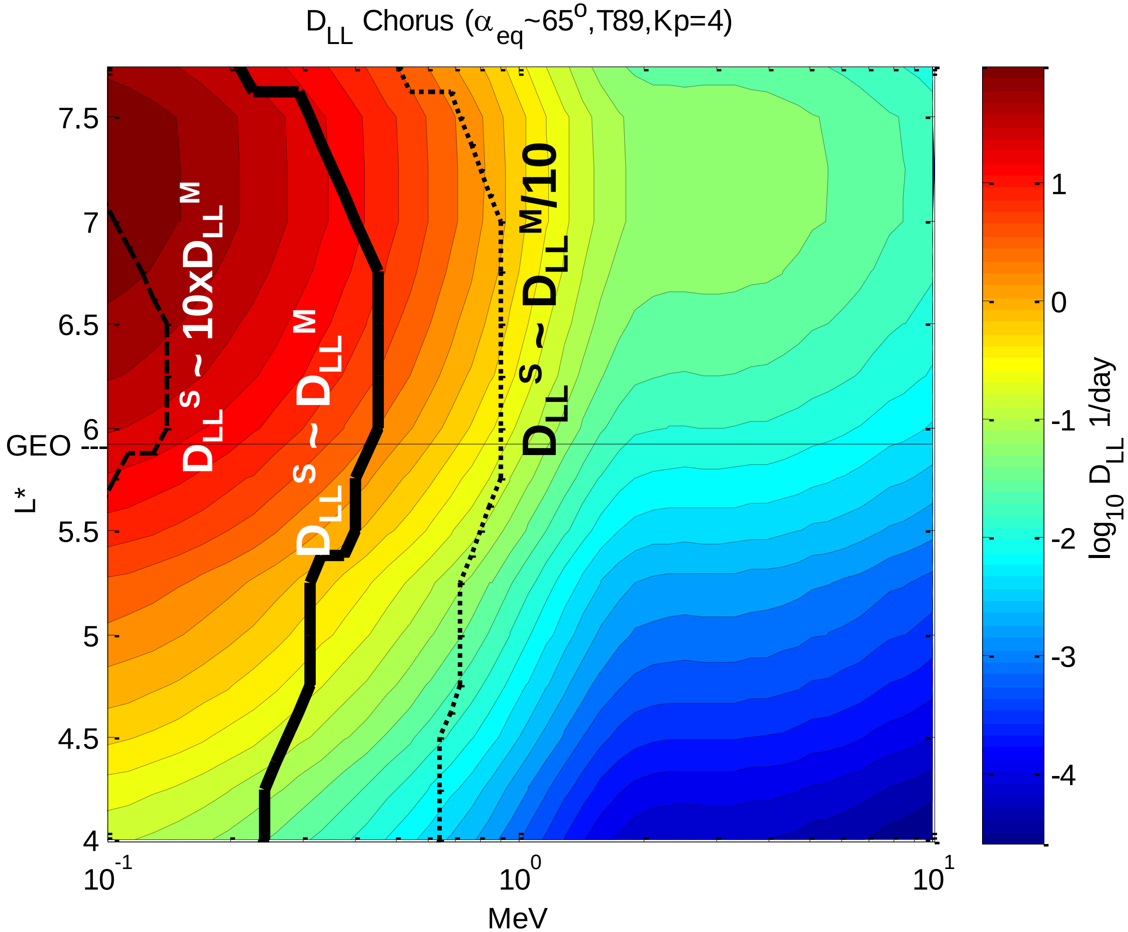
- Pitch-angle diffusion is strongest for E<~300 keV and α_{eq} < ~70
- We scaled $D\alpha_{eq}\alpha_{eq}$ to other *L* values as $(L/4.5)^3$

Magnetic shell splitting



- Using T89 with Kp=4, we computed $dL^*/d\alpha_{eq}$ for a particle with $\alpha_{eq} \sim 65^o$.
- As expected, drift shell splitting is “large” ~ 1/rad in the region near and beyond GEO
  - *On the night side, higher pitch angles correspond to higher L**
  - *On day side, higher pitch angles correspond to lower L**
  - *At dawn and dusk,* $dL^*/d\alpha_{eq} \sim 0$
- Splitting has been inferred from “butterfly” pitch angle distributions at GEO

Estimated magnitude of anomalous radial diffusion



- For comparison, we computed the magnetic diffusion coefficient D^M_{LL} according to *Brautigam and Albert* [2000]
- We estimate that anomalous radial diffusion is important <~300 keV (stronger diffusion) and near GEO (stronger splitting)

- Anomalous (cyclotron-resonant) radial diffusion can be as large as 10 times drift-resonant magnetic diffusion

Conclusions

- Cyclotron-resonant pitch-angle scattering can lead to anomalous radial and cross diffusion on split magnetic drift shells
- We have shown that anomalous radial diffusion is likely significant at L shells near GEO and energies below ~300 keV
- The same reasoning implies the presence of significant new cross terms, especially D_{ML} and D_{KL} which would require alteration of many 3-D diffusion codes which only address D_{MK} cross terms
- While data analysis has been accounting for the non-dipole field for over a decade, it now appears time for diffusion simulations to do so as well

- Note: Codes supporting these calculations can be found in the IRBEM Lib at http://irbem.sourceforge.net in the "Extras" section, called "Open Diffusion Code"