ULF wave interactions with energetic radiation belt particles

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(for a less hyperkinetic version of the material presented in this talk, please see "A review of ULF interactions with radiation belt electrons" by S.R. Elkington, in *Magnetospheric ULF Waves*, AGU Monograph 169, 2006.)

RBSP SWG Team Meeting
JHU/APL
February , 2013

Understanding radiation belt dynamics: transport in invariant space

Wave particle interactions lead to transport in invariant space, described by the Fokker-Planck equation:

$$\frac{df}{dt}+\frac{1}{\Im}\sum_i \frac{\partial}{\partial J_i}\left[\Im\left\langle\frac{dJ_i}{dt}\right\rangle f\right]=\frac{1}{\Im}\sum_{i,j}\frac{\partial}{\partial J_i}\left(\Im D_{ij}\frac{\partial f}{\partial J_j}\right)-\frac{f}{\tau}+S$$

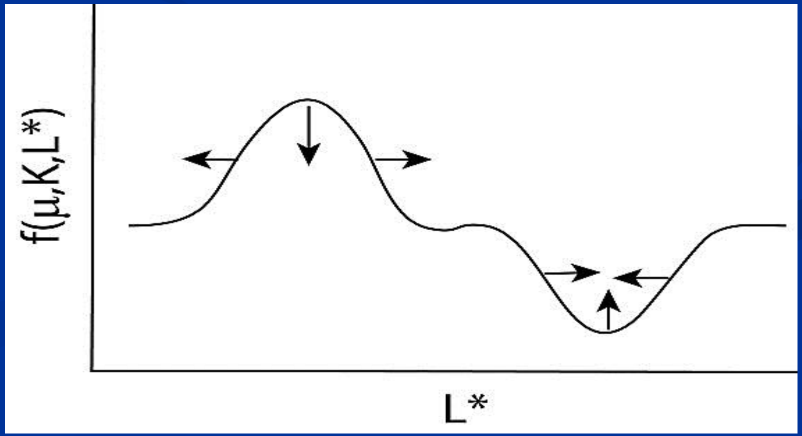
Sources

Advective terms

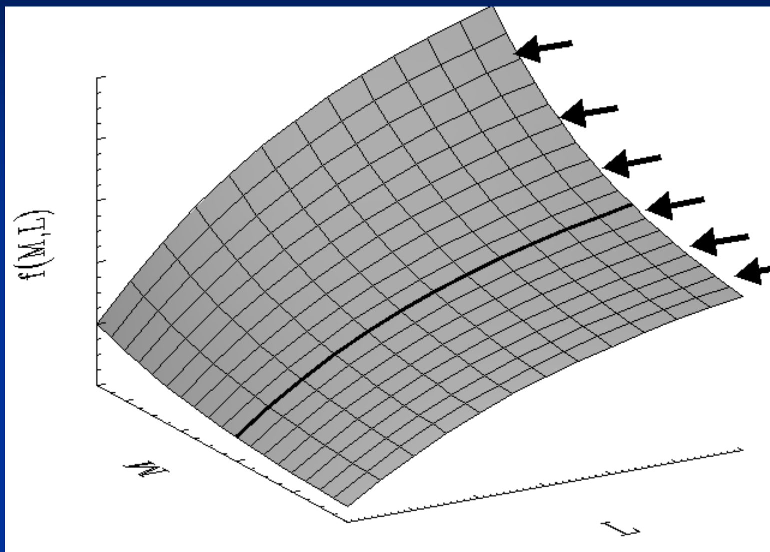
Stochastic terms

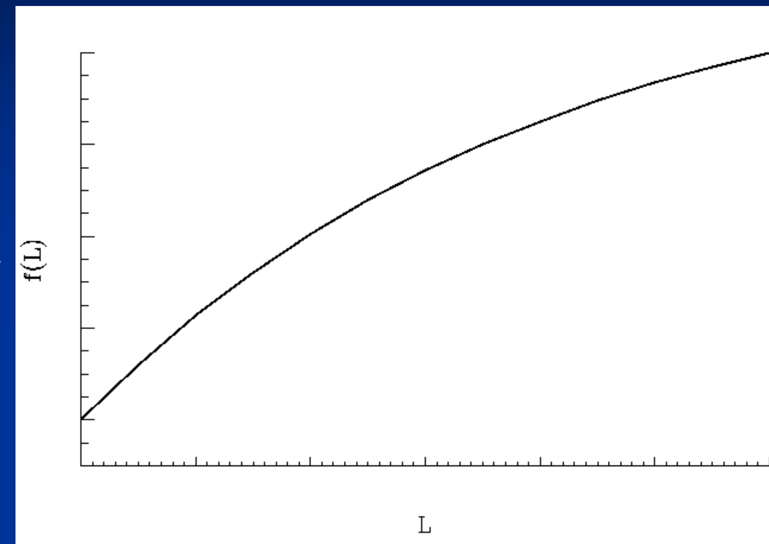
Losses

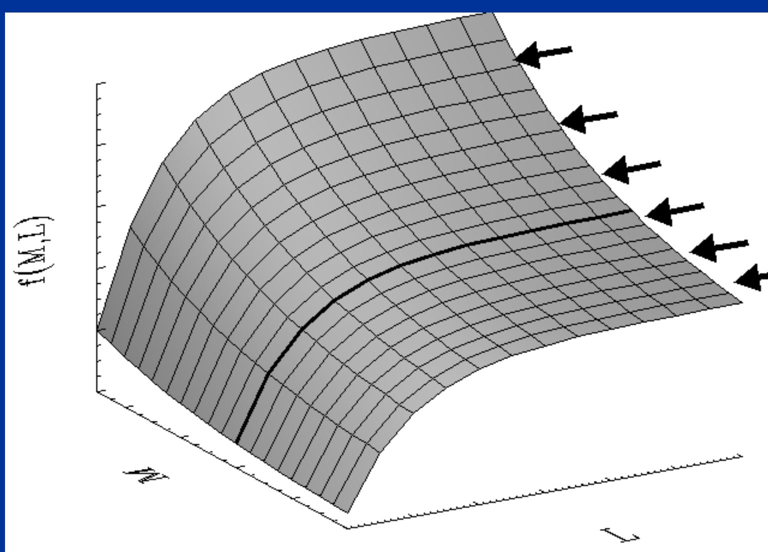
Diffusive processes will tend to smooth gradients in a particular coordinate in accordance with "Fick's Law".

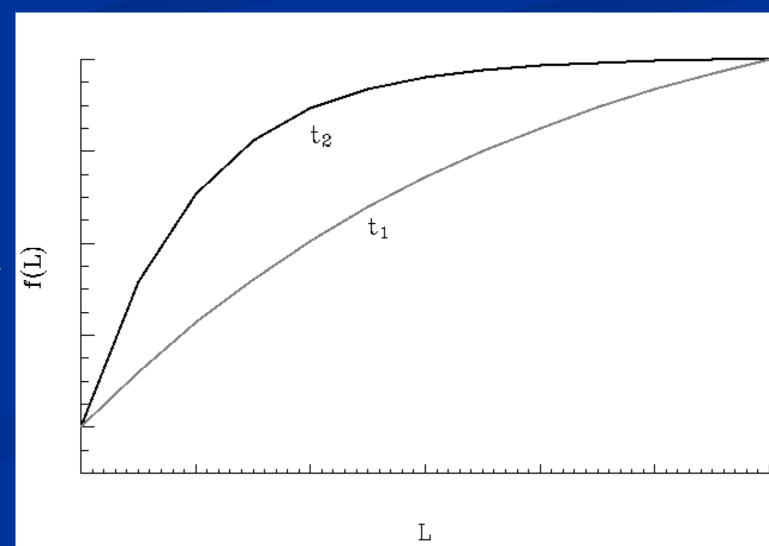


Transport in *L*: radial transport

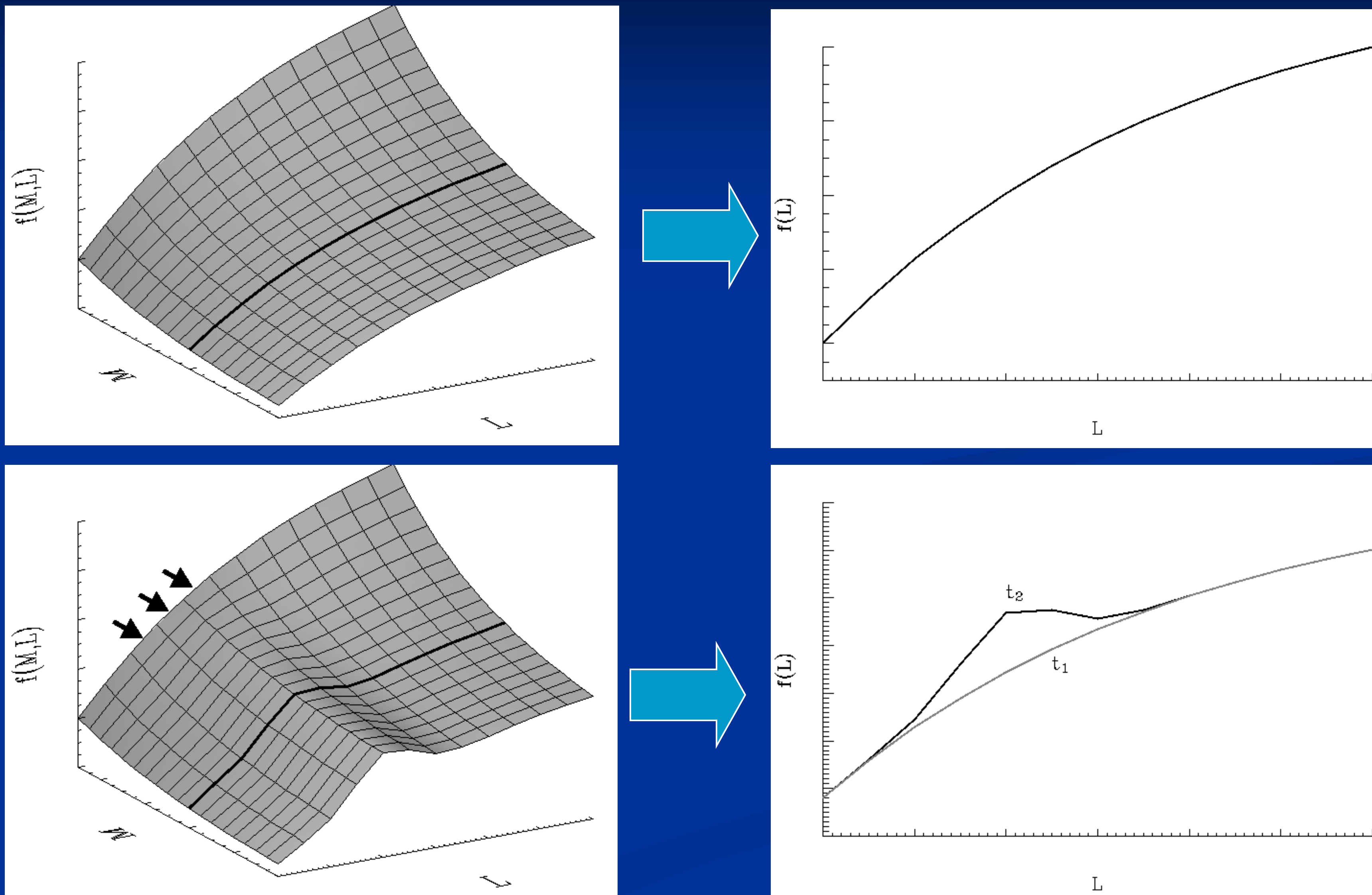








Transport in M, K: local heating



Adiabatic vs non-adiabatic acceleration via ULF waves

Energetic particle interactions with ULF waves may be categorized as Adiabatic vs non-Adiabatic, according to whether they conserve their first invariant in the interaction.

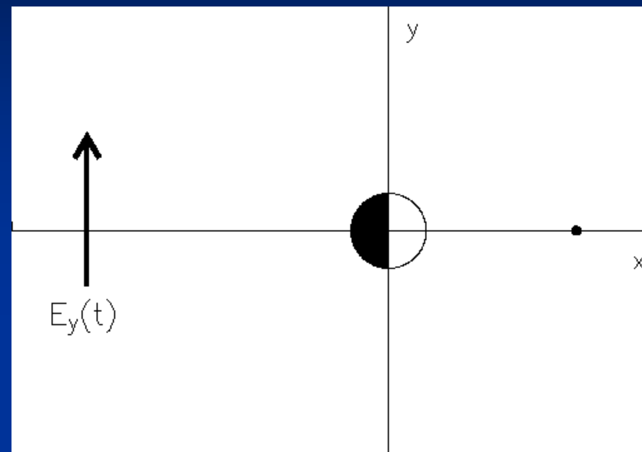
(At least) two non-adiabatic interactions have been proposed:

- Liu et al., 1999:
  - Particles interact with global, symmetric (m=0) ULF modes.
  - Pitch angle scattering leads to differing drift rates, and hence energization.
  - Bulk energization via 'zero energy boundary', most effective at small pitch angles and when $T_{scat} \sim T_{ULF}$
- Summers and Ma, 2000: transit-time acceleration
  - Fermi acceleration process based on gyroresonant interactions with oblique fast mode turbulence.
  - Interaction requires $T_{ULF} \sim \lambda_{||}/v_{||}$

Adiabatic energization and Pc5 ULF waves

Energy of an electron moving in a dipole magnetic field with slowly-varying dawn-dusk convection electric field

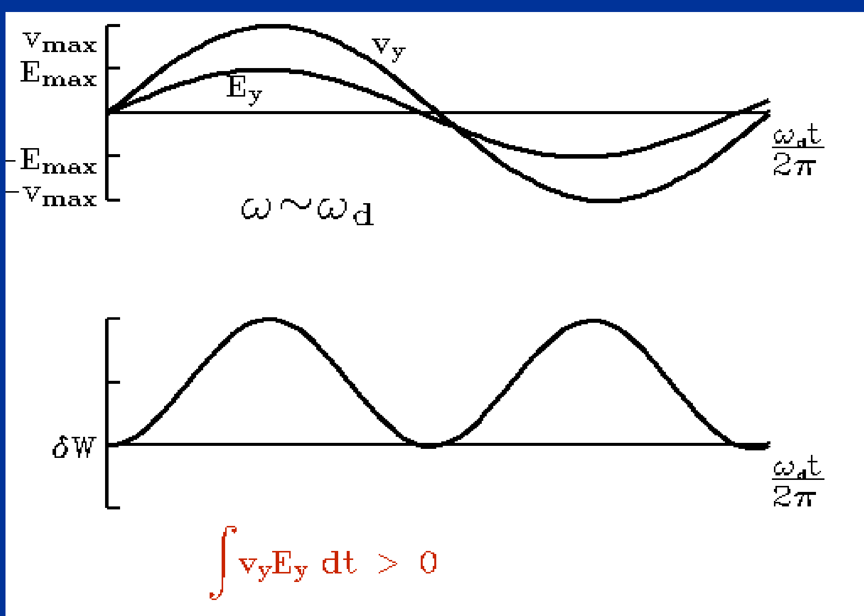
$$\frac{dW}{dt} = q\,\vec{v}_D \cdot \vec{E} + \frac{M}{\gamma}\frac{\partial B}{\partial t}$$

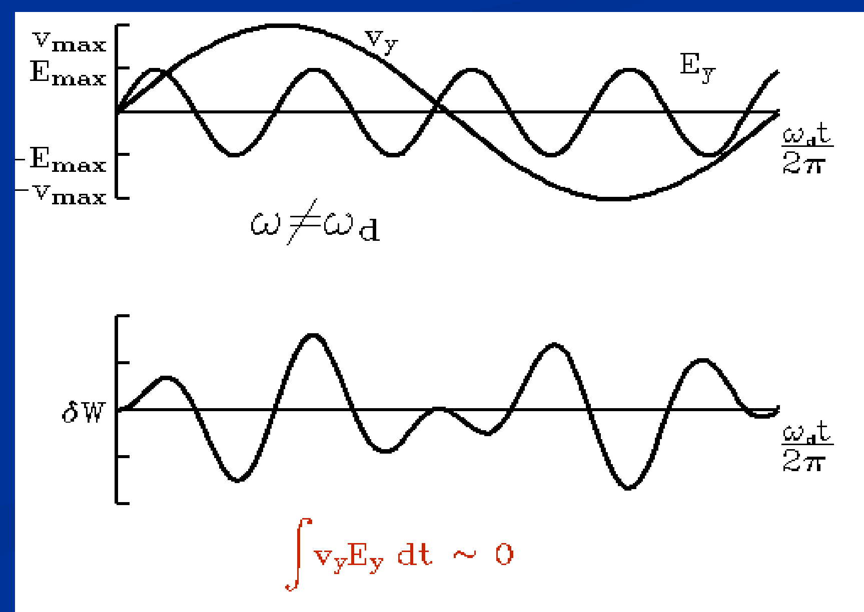


- f_D~mHz ⇨ ULF waves.
- Only need consider resonant frequencies

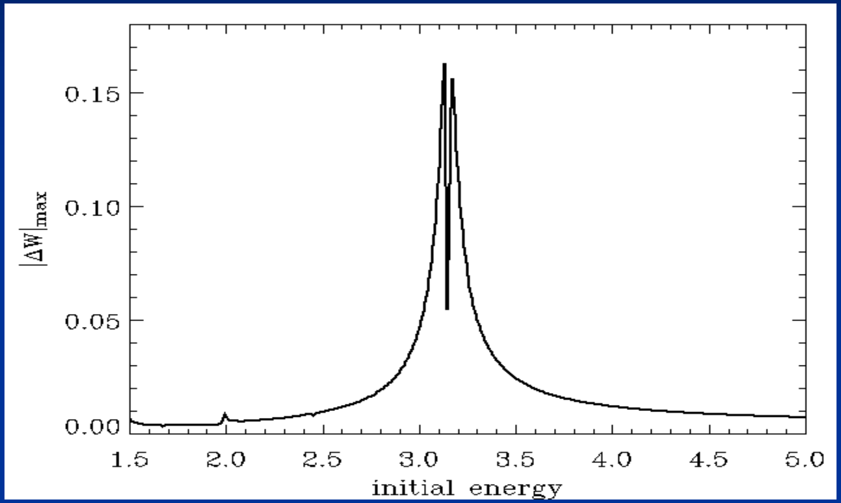
$$E = E_0 \cos(m\varphi - \omega t + \xi)$$

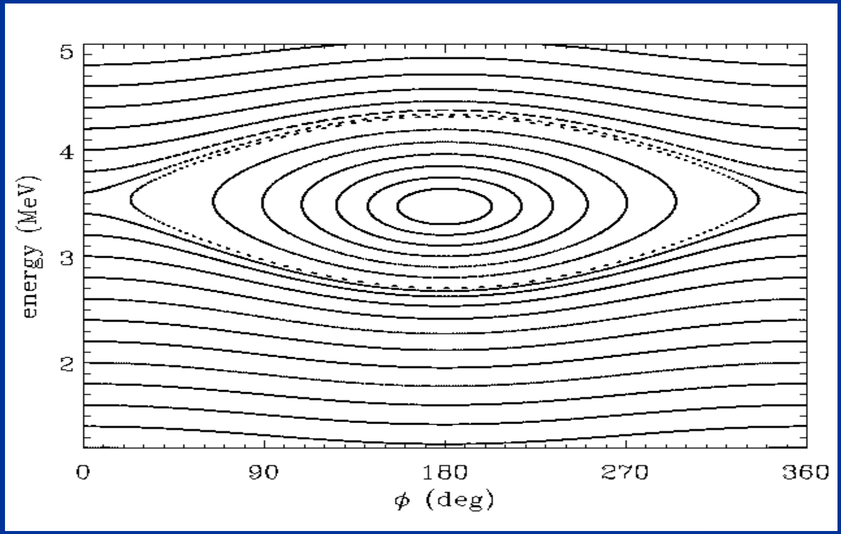
$$\omega = m\omega_D$$

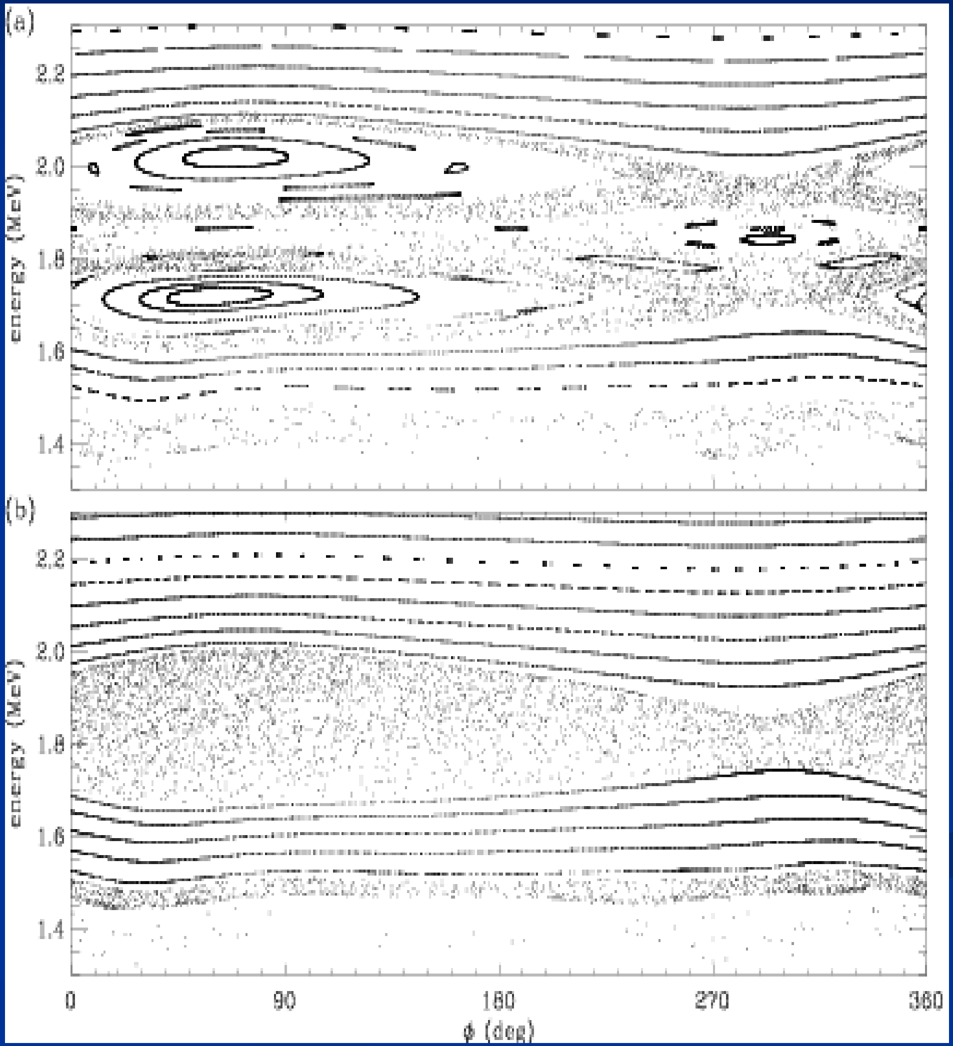




Diffusion resulting from resonant wave-particle interactions







Partitioning of the electric field

Electrostatic Variations: $\nabla \times E_{conv} = 0$

$$D_{LL}^{E} = \frac{1}{8B_0^2 R_E^2} \cdot L^6 \cdot \sum_m P_m^E (L, m\omega_D)$$

Electromagnetic Variations: $\nabla \times E_{ind} = -\partial B / \partial t$

$$D_{LL}^{B} = \frac{1}{8}\left(\frac{M}{q\gamma B_0 R_E^2}\right)^2 \cdot L^4 \cdot \sum_m P_m^B (L, m\omega_D)$$

$$E_{meas} = E_{ind} + E_{conv}$$

•The power P^B in the electromagnetic diffusion coefficent is actually the sum of two parts: power in the time varying magnetic field, and the power in the resulting induced electric field.

•Falthammar (e.g. 1965) showed that the effect of each of these diffusive fields had approximately equal effect: 7/15 of the effect comes from the time-varying magnetic field, and 8/15 from the induced electric field.

•Given a measured electric field E $(=E_{conv}+E_{ind})$, how do we know how much E should be applied to $D_{LL}{}^{E}$, and how much is already accounted for in $D_{LL}{}^{B}$?

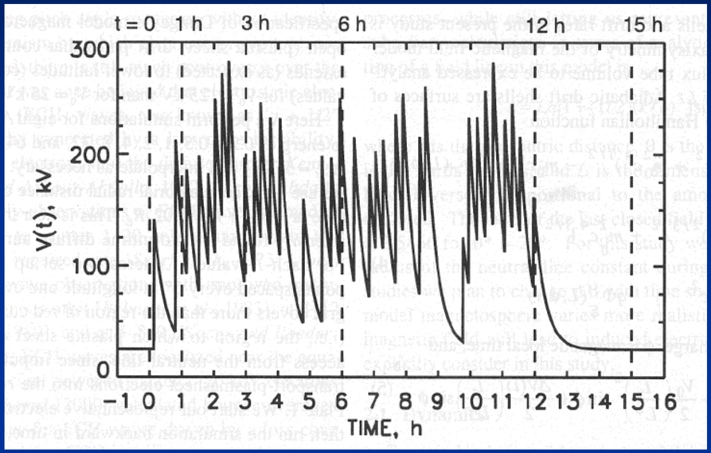
Particle energization and ULF waves

- Magnetospheric ULF waves driven by the solar wind can effectively heat and transport energetic particles in geospace.
- Efficiency of particle energization depends on explicit physical properties of the induced waves.
    - ✓Frequency spectrum of the waves
    - ✓Radial extent of the waves
    - ✓Azimuthal propagation of the waves
    - ✓Azimuthal extent of the waves
    - ✓Mode structure of the waves
    - ✓Electromagnetic character of the waves

$$\omega = m\omega_D$$

- Mode structure, spatial extent, and propagation characteristics are not easily obtained from single-point measurements of ULF waves.
- Lacking necessary in-situ measurements, global simulations (e.g. MHD) may be able to provide the missing information.

Empirical models of diffusion: Brautigam & Albert *[JGR 2000]*



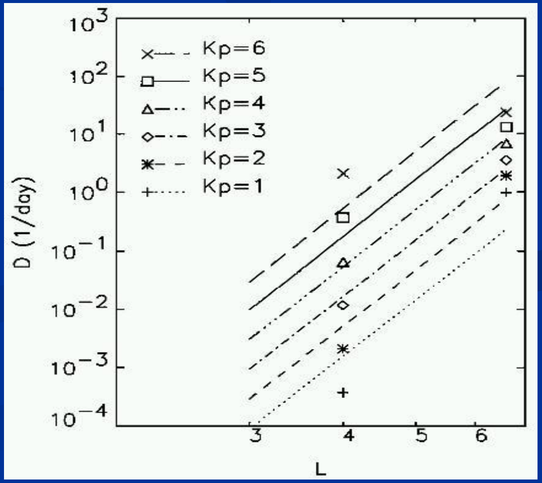
- Assume electric field fluctuations with fast rise and exponential decay, resulting P^E gives D_{LL} of form

$$D_{LL}^{E} = \tfrac{1}{4} \cdot \left(\frac{cE_{rms}}{B_0} \right)^2 \left(\frac{T}{1 + (\omega_D T/2)^2} \right)$$

with

$$E_{rms} = 0.26(K_p - 1) + 0.1\,\mathrm{mV/m}$$

based on assumption that E_{rms}=0.1 mV/m when quiet *[Lyons & Thorne, 1973]* and E_{rms}=1.4 mV/m when disturbed *[Lyons & Schulz, 1989]*.



- Fitting D_{LL} for magnetic fluctuations observed by balloon at L=4.0 and 6.6 *[Lanzerotti et al., 1978]* to L^{10} functional form gives

$$D_{LL}^{B} = 10^{(0.506\,K_p - 9.325)}\,L^{10}$$

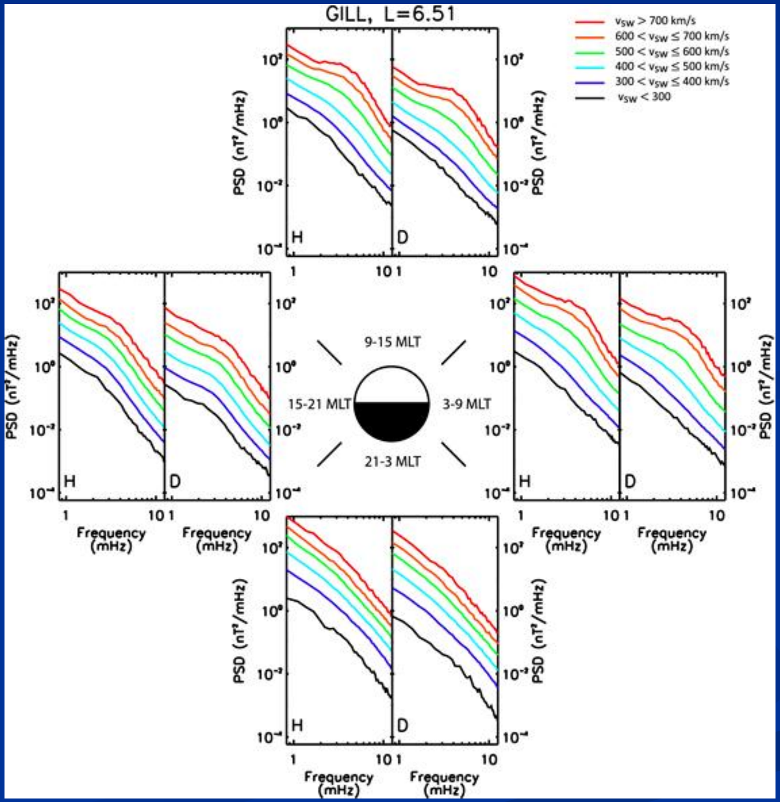
- By these formulae, $D_{LL}{}^{B}$ dominates $D_{LL}{}^{E}$.

Other (better?) parameterizations of the ULF wave environment

- Brautigam et al. *[JGR 2005]* used CRRES data to get a better parameterization of the electric field power:

$$P^E = aL^b \exp\left(cK_p\right)$$

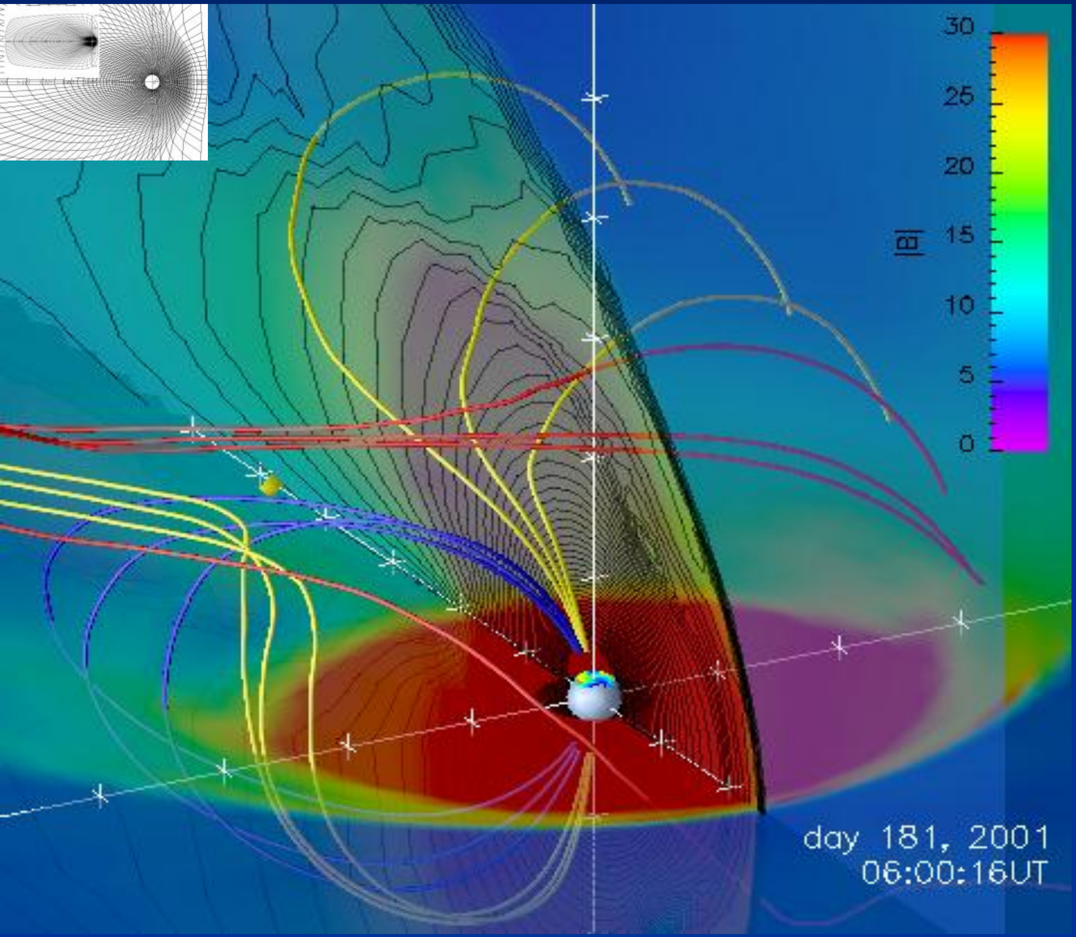
- Rae et al. *[JGR 2012]* used ground magnetometers and an Alfvenic fluctuation model to determine the electric power spectral density in space, and found good agreement with Brautigam et al. *[JGR 2005]*



- Ozeke et al. *[JGR 2012]* similarly used ground magnetometers to generate $D_{LL}{}^{E}$, and in situ measurements (GOES, AMPTE) to generate $D_{LL}{}^{B}$.

In contrast to Brautigam & Albert, the Ozeke et al. results suggest $D_{LL}{}^{B} << D_{LL}{}^{E}$.

Test particle simulations with physically-based field models: MHD

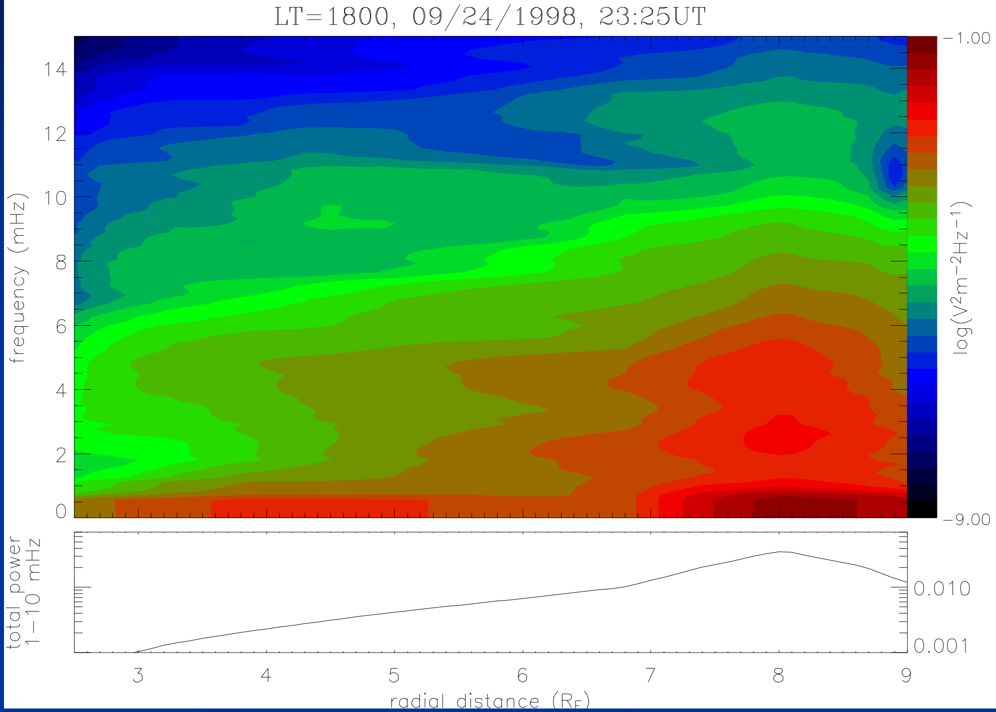


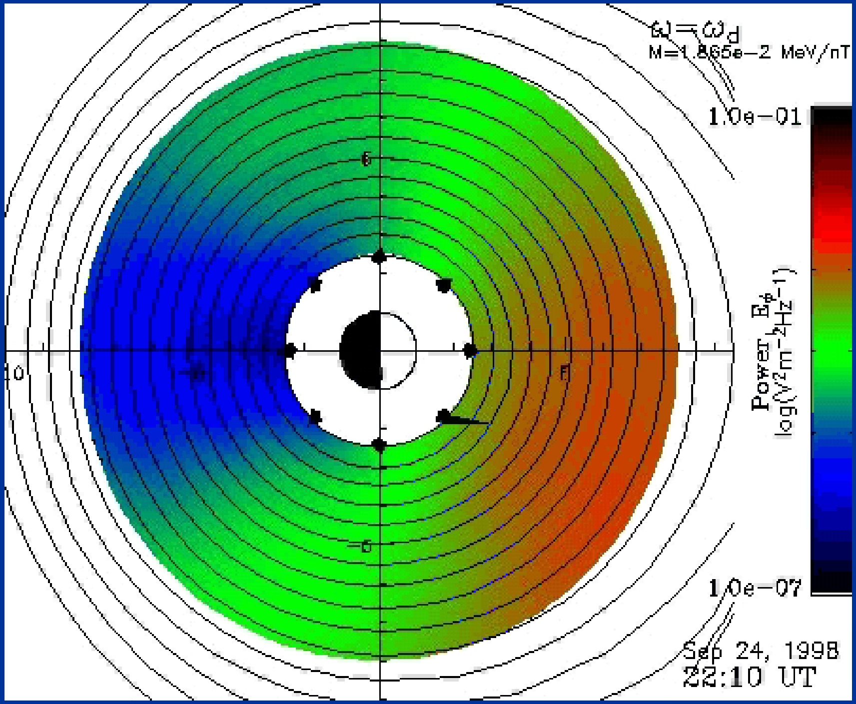
- MHD treats magnetospheric plasma as magnetized fluid
- Driven by upstream boundary conditions
- Includes reconnection, convection, external contributions to B, etc.
- Does not include
  - High frequency waves
  - Multiple plasma species
  - Kinetic effects
  - Hall physics
  - Etc…

The Lyon-Fedder-Mobarry is under active development via the NSF Science Technology Center (STC) program and the *Center for Integrated Space Weather Modeling*, a 10-year, $40M program designed to construct an integrated physical model of the entire Sun-Earth system.

Spectral analysis: general characteristics

Analysis first conducted at series of points within magnetosphere to discern general characteristics of wave activity.





- ULF power in MHD shows more power at high *L*, low *f*.
- Spectral power indicates structure in azimuth.

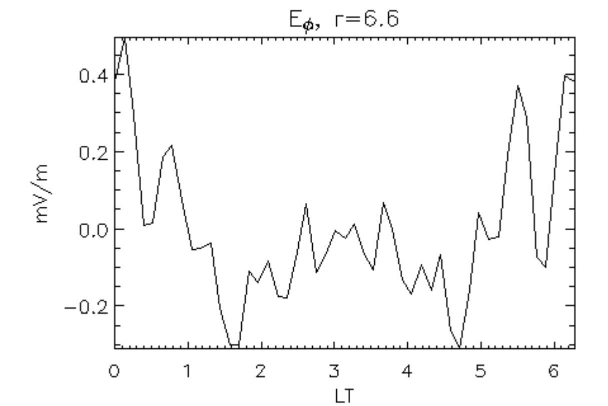
Mode structure: spectral analysis

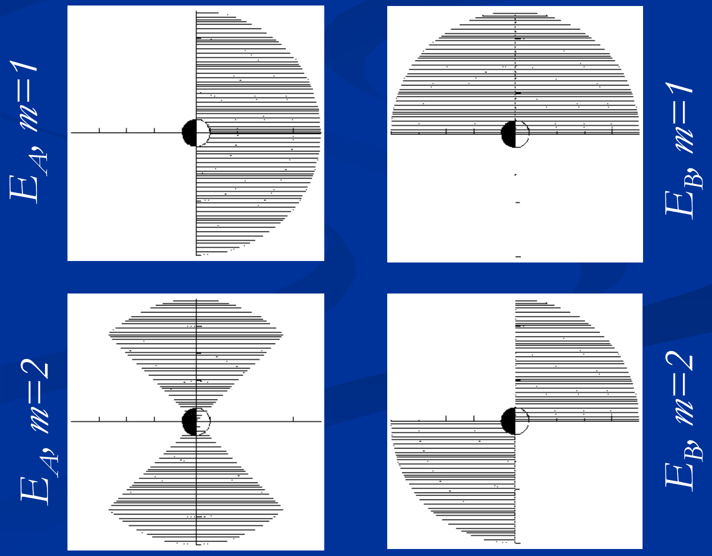
The resonant condition driving radial diffusion is given by $\omega = m\omega_D$, where m is the azimuthal mode number of the waves. We thus need the power as a function of both frequency, ω, and m.

We follow the technique proposed by Holzworth and Mozer (1979):

- Fields Fourier analyzed in azimuth at each radial distance to obtain azimuthal mode coefficients E_A, E_B for each mode number m.
- Mode coefficients Fourier analyzed in time to obtain power in each mode number.
- Total power is

$$P_m = P_m^A + P_m^B$$





Mode structure of ULF waves

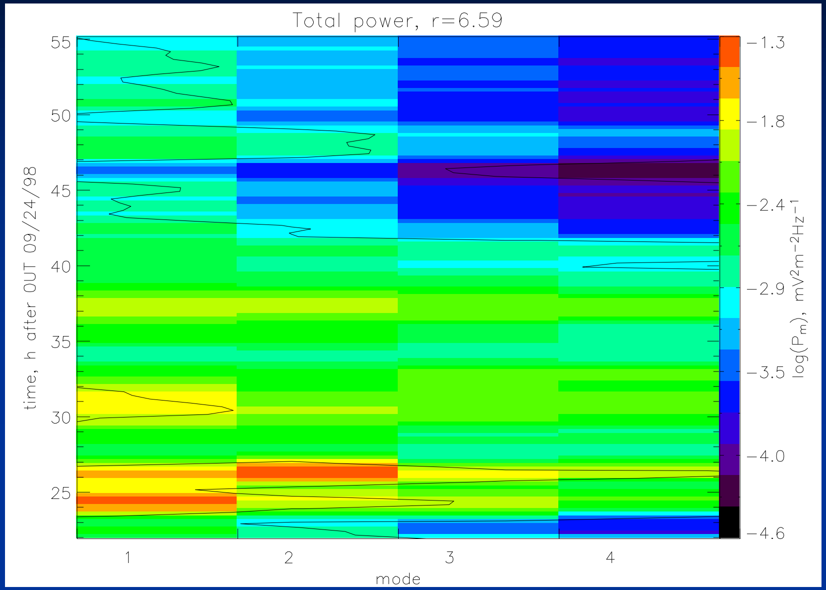
- The CME-driven storm in this study had a (typical) extended period of southward IMF during the main phase of the storm.
- The HSSW-driven storm had a (typically) fluctuating IMF during main phase.
    - The CME storm exhibited a relatively large amount of power during main vs recovery phase.
    - The HSSW storm was not so clear; if anything, there was more power during recovery phase when the solar wind speed was higher.

➢ *IMF orientation has some effect on ULF generation/growth, but the contrasting effects of B_z vs V_{sw} has not been addressed in this study.*

CME

Recovery

Main Phase

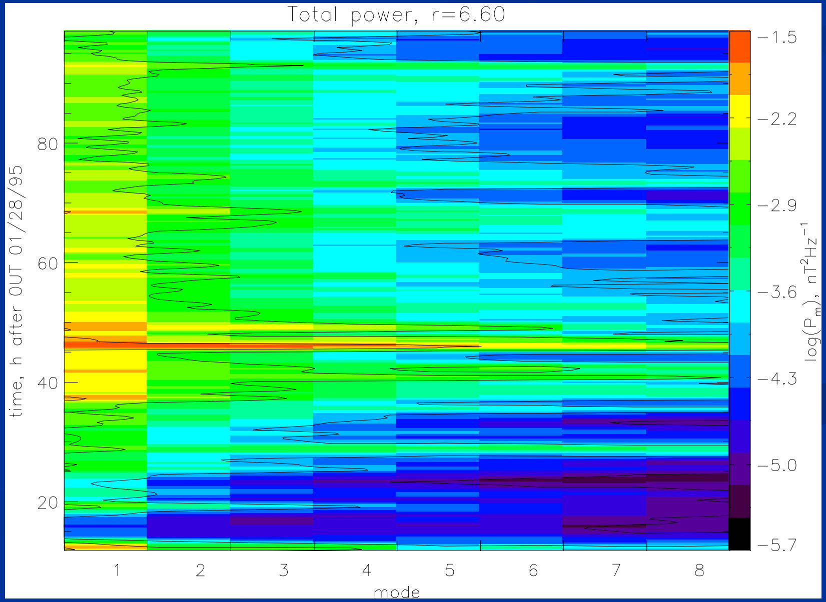


September 1998

HSSW

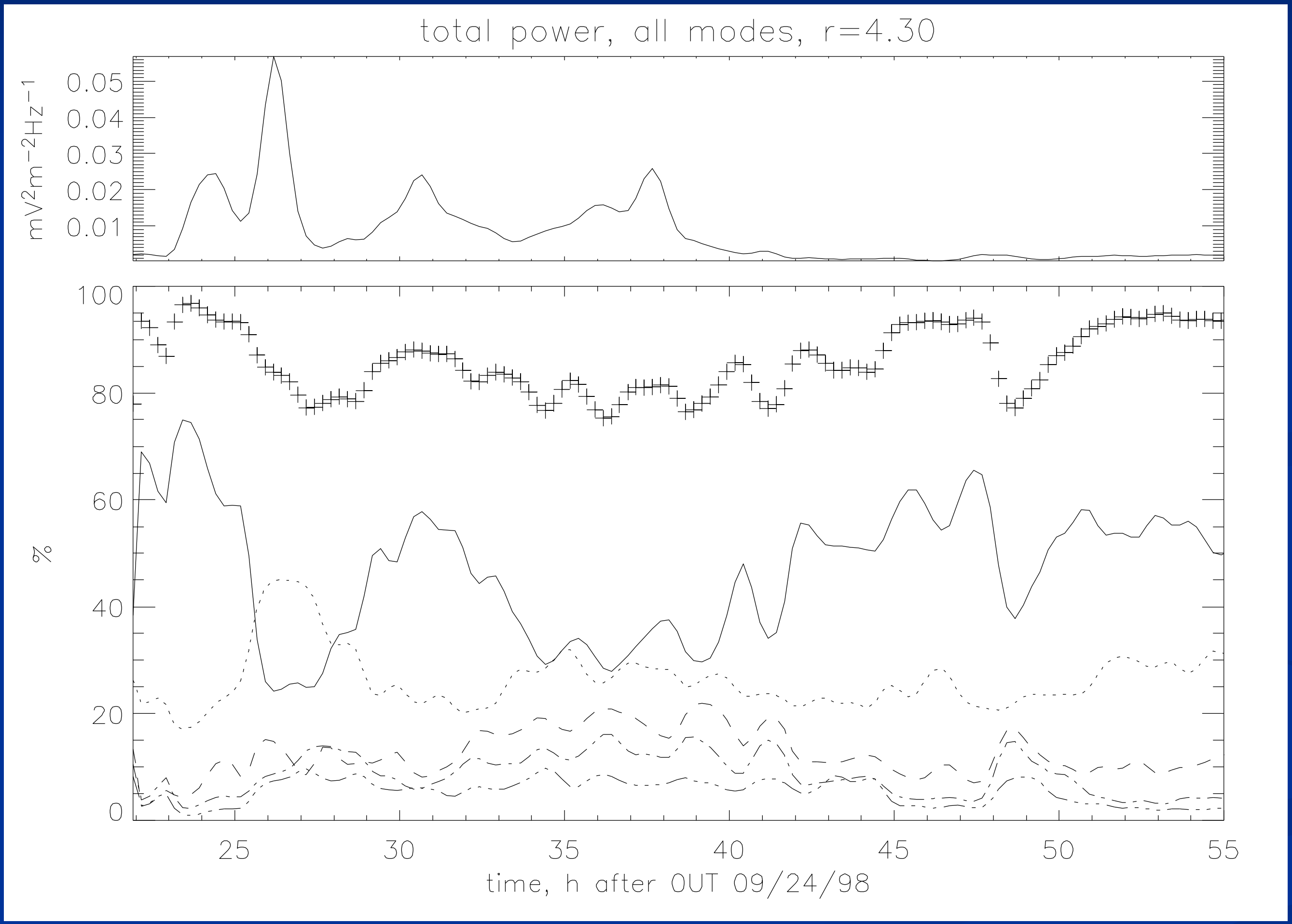
Recovery

Main Phase



January 1995

How much power is in each mode?



Total power in m=1-5

$P(m=1+2)/P(m=1..5)$

$P(m=1)$

$P(m=2)$

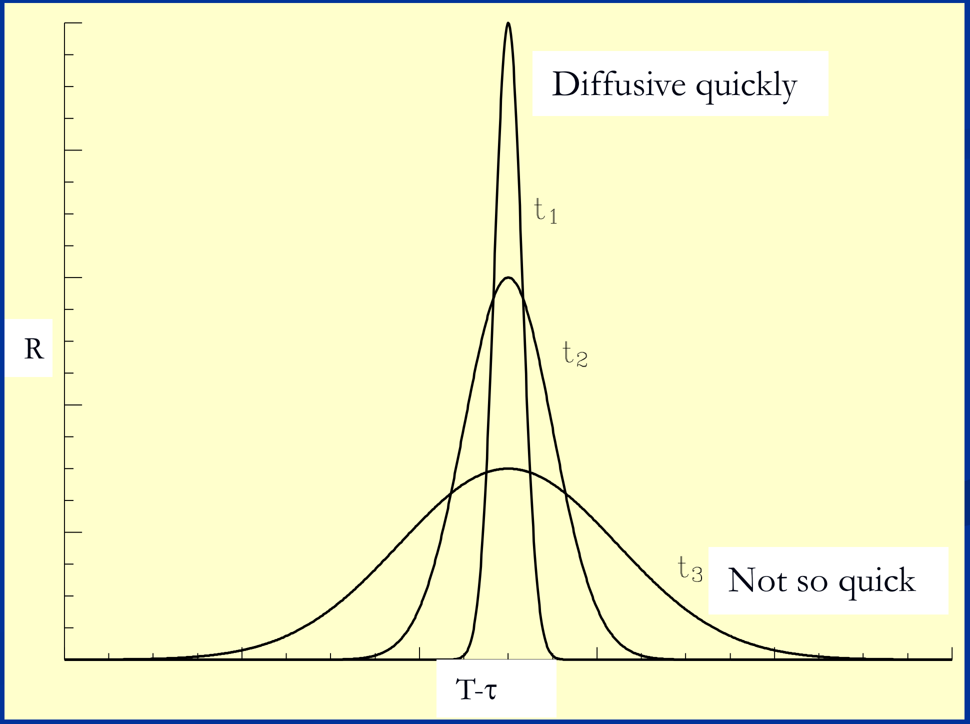
$P(m=3\text{-}5)$

Diffusion time scales

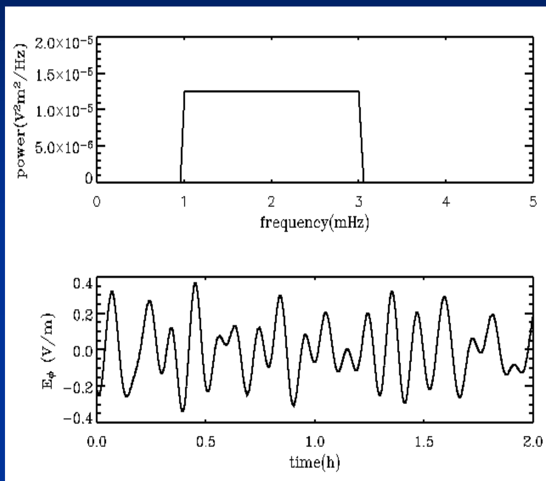
- An important question regarding the nature of radial diffusion is *"what is the minimal time scale over which the wave-particle interaction may be considered stochastic?"*
- Previous efforts have only stated the requirement that $T >> T_D$.
- We are trying to calculate this minimum scale in terms of a diffusive autocorrelation time,

$$R(\tau) = \langle v_r(t) \cdot v_r(t-\tau) \rangle$$

- We will show here that the autocorrelation time also has an important effect on the effect of waves occurring over a limited local time extent.



Quantifying radial transport via test particle simulations



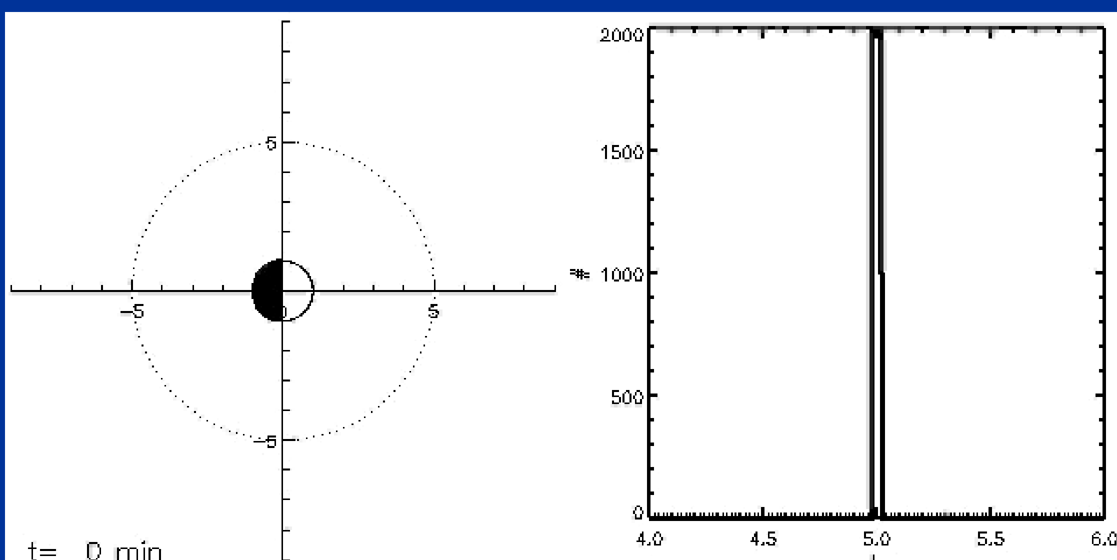
Scenario:

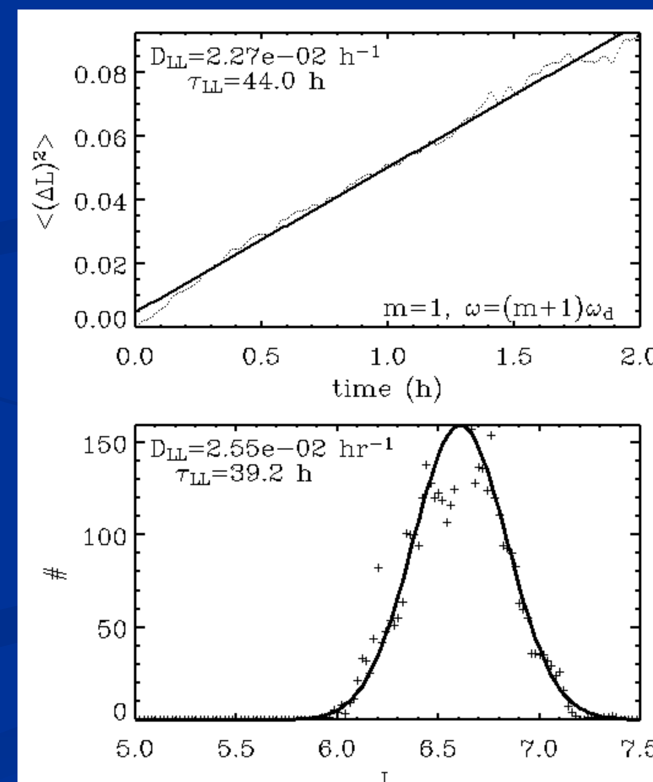
•Ensemble of particles initially at *L*=5 in a dipole field.

•Dynamic waves: analytic ULF with frequencies $\sim f_d$ and random phases induce radial diffusion.

Quantifying diffusion:

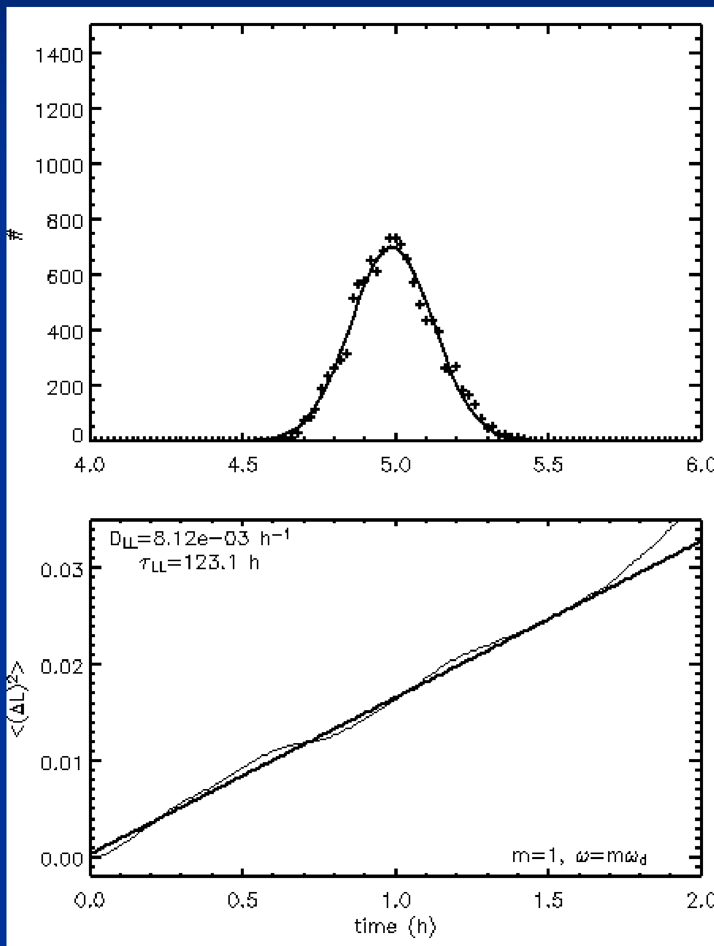
$$D_{LL} = \frac{\left\langle (\Delta L)^2 \right\rangle}{2\tau}$$





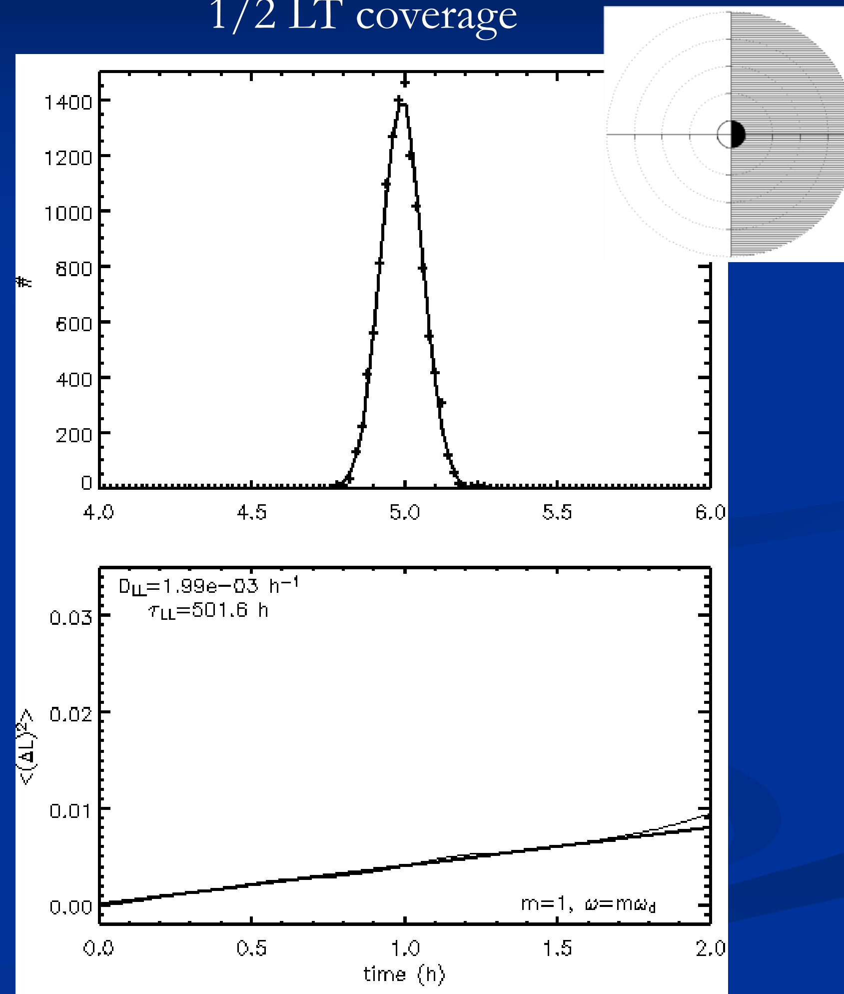
Azimuthal extent of the waves

Global waves



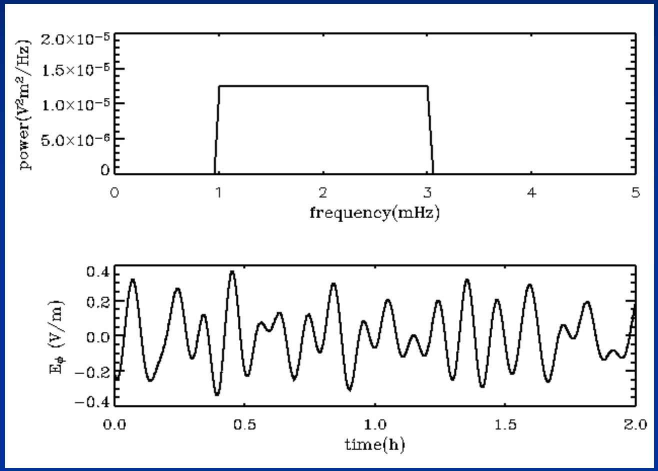
$D_{LL}=D_{REF}$

1/2 LT coverage

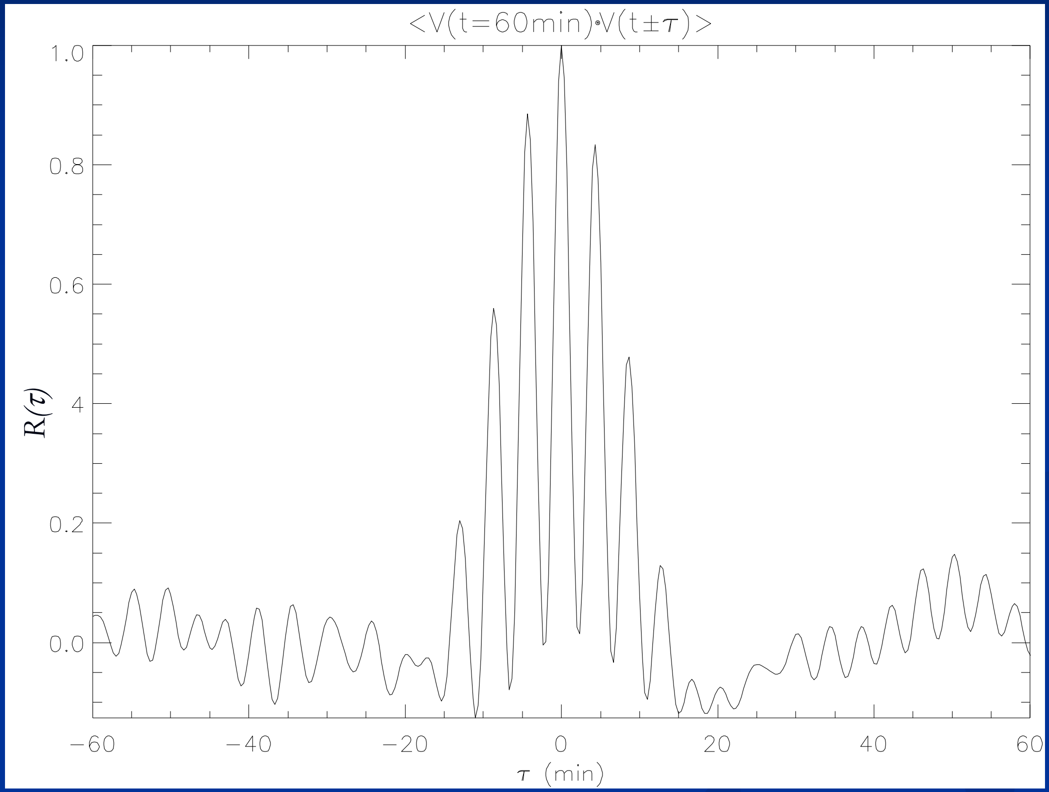


$D_{LL}=D_{REF}/4$

Autocorrelation calculation: relation to driving ULF wave function?

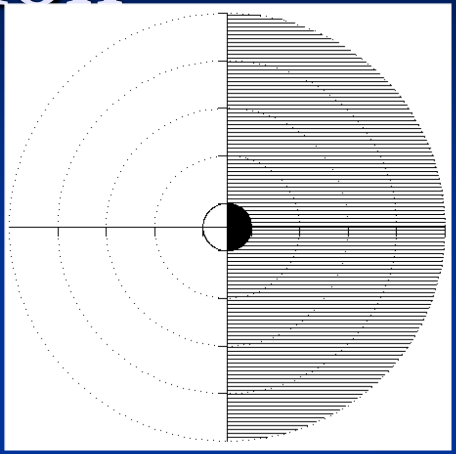


A ring of particles at *L*=6 evolve under the influence of a continuum of waves with constant PSD over an interval *Δf*.

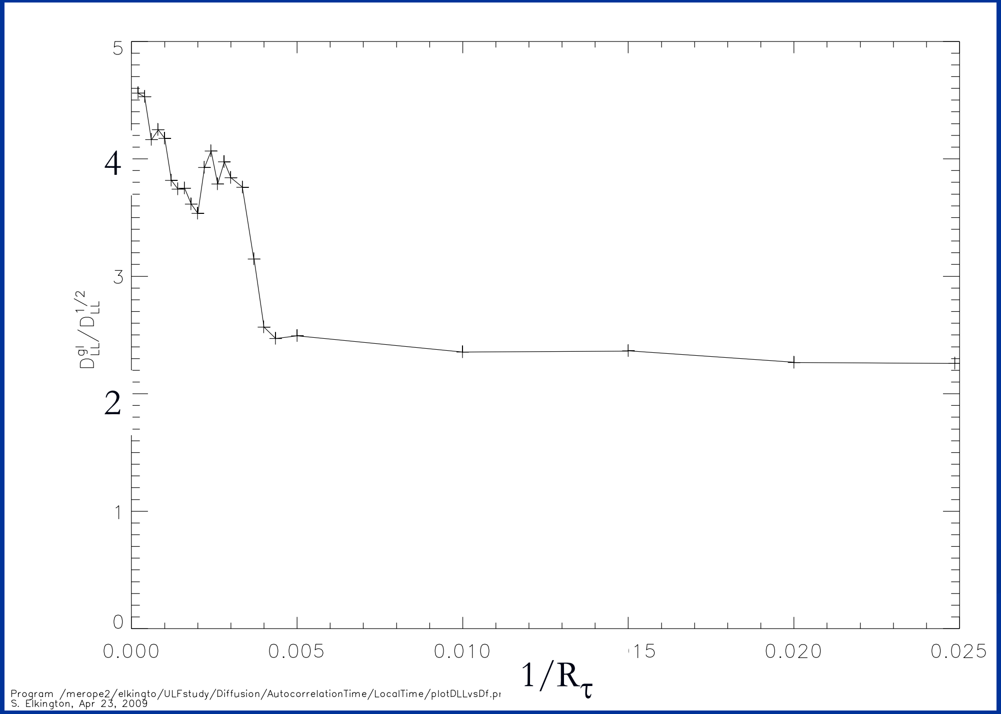


By definition, the power spectral density is the Fourier transform of the autocorrelation function. The Fourier Transform of a square wave function of width 2L goes like sin(Lt)/Lt, so the *power distribution of our driving ULF waves may account for the functional form of our calculated autocorrelation relation.*

Autocorrelation times, local time effects, and diffusion



•For large autocorrelation times, diffusion rates appear to be dictated by the amplitude of the waves in the region over which they occur.
•For smaller autocorrelation times, diffusion rates appear to be dictated by the power of the waves in the region over which they occur.



To understand the effects of ULF waves in a limited local time in the magnetosphere, it appears to be necessary to understand autocorrelation times for ULF waves in the magnetosphere.

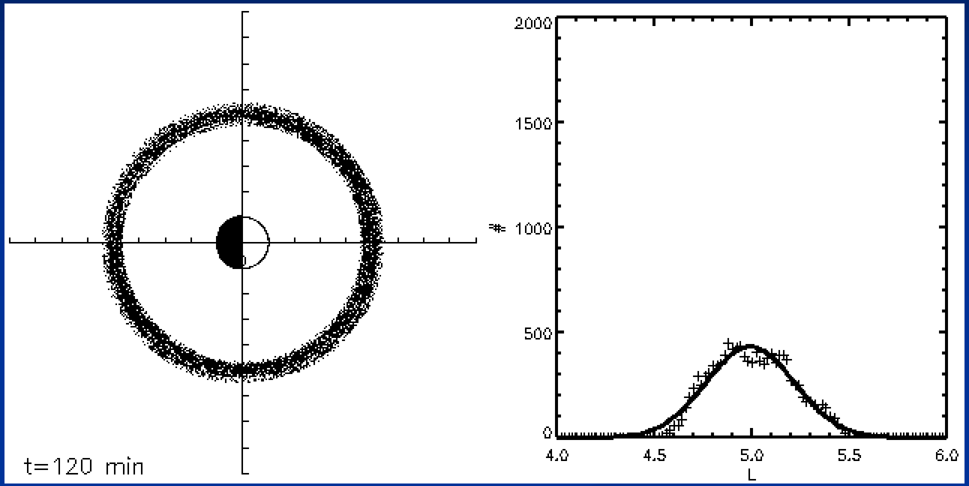
Presumably, this is related to the autocorrelation time for variations in the solar wind.

Is radial transport diffusive?

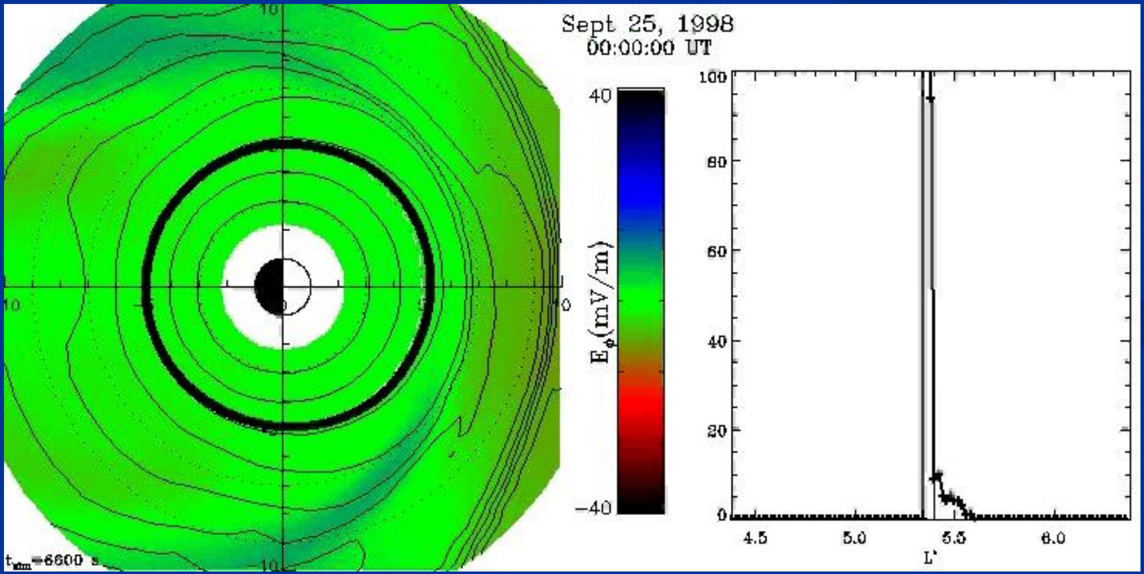
Global MHD simulations allow us to do a detailed analysis of wave power as a function of frequency, radial location, local time, mode number, etc.

In principle, we should be able to compare test particle simulations to quasilinear diffusion theory to investigate these effects.

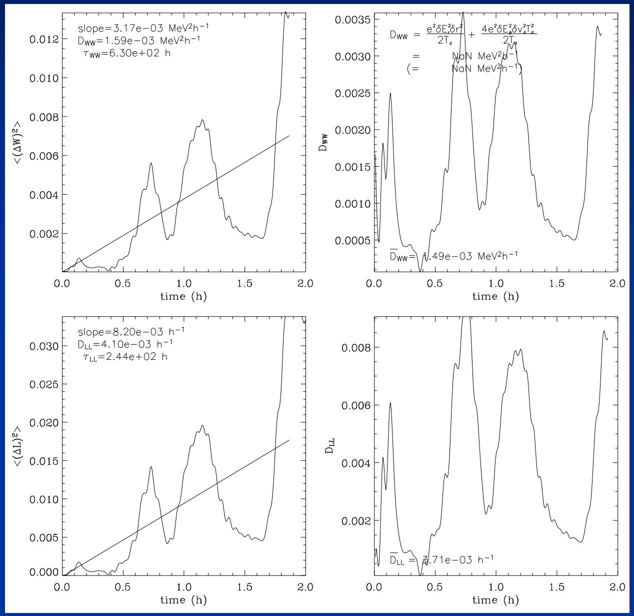
We expect:



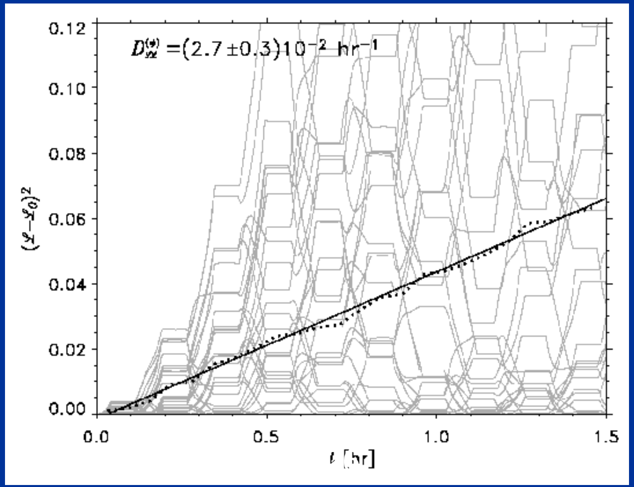
We get:



Diffusion may only be a viable description of radial transport in an 'average' sense…



Elkington et al.



Ukhorskiy et al.

Comparison of Diffusion and Particle Drift Descriptions of Radial Transport in the Earth's Inner Magnetosphere

Pete Riley and R. A. Wolf

Department of Space Physics and Astronomy, Rice University, Houston, Texas

A comparison is made between two approaches, radial diffusion and guiding-center drift, for describing the radial motion of charged particles in the equatorial plane of the Earth's magnetosphere. For the storm event of August 1990, a time-dependent, observation-based model electrostatic field is computed. This field is then used (1) to calculate the drift of a ring of monoenergetic test particles and (2) to compute the specific radial diffusion coefficient and solve the relevant diffusion equation. Because of the electric field model employed and the data used to construct it, the particles considered were restricted to those with drift periods greater than 2 hours and therefore energies less than about 130 keV at L=3. Thus our calculations bear directly on only the low-energy part of the radiation belts. Density profiles computed from diffusion theory are compared with the results of the guiding-center simulation for a number of initial conditions. Mediocre agreement is found when the diffusion results are compared with the guiding-center simulation for the August 1990 event. However, for cases where (1) a number of electric fields are recovered from the power spectrum and the particle drifts are averaged or (2) a number of electric fields are applied sequentially to a single distribution, the agreement is considerably better. The main conclusion from these tests is that the diffusion formalism gives only roughly right answers for a single real storm but does much better for an average over a statistical ensemble of storms. Finally, several previously derived diffusion coefficients are compared with the present one as functions of energy.

Riley and Wolf (*JGR, 1994*): "The main conclusion from these tests is that the diffusive formalism gives only roughly right answers for a single real storm, but does much better on average over a statistical ensemble of storms."

Summary and Open Questions

- PC5 ULF waves may affect radiation belt particles either by direct (local) heating, or through adiabatic transport
- For adiabatic transport (aka radial diffusion), we need to properly characterize the waves activity:
  - What is the power spectral density of the waves?
    - What is the extent of this power in local time?
    - How does this power vary with radial distance?
  - What is the mode structure of the waves?
  - What is the direction of propagation of the waves?
  - Is it possible to distinguish between induced electric fields (intrinsic in D_{LL}^{B}) and convection electric fields (explicit in D_{LL}^{E})?
  - What are relevant time scales over which radial transport appears ‘diffusive’?
  - Is ‘radial diffusion’ the proper way to describe radial transport in the belts?